Securely Instantiating `Half Gates' Garbling in the Standard Model

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Dennis Hofheinz	ETH Zurich	
Chethan Kamath	IIT Bombay	

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AARHUS UNIVERSITET

GARBLING SCHEMES

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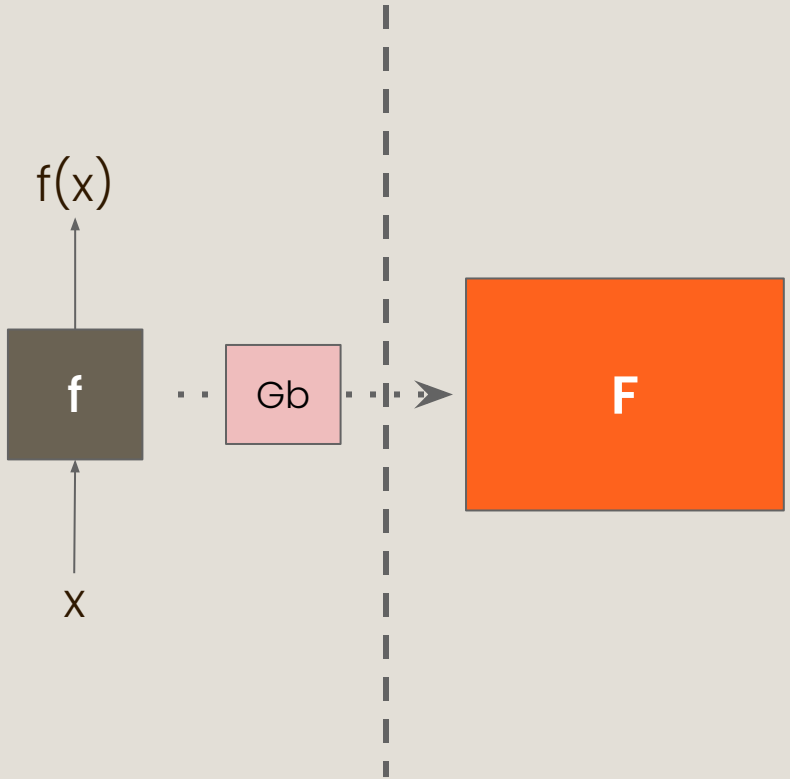

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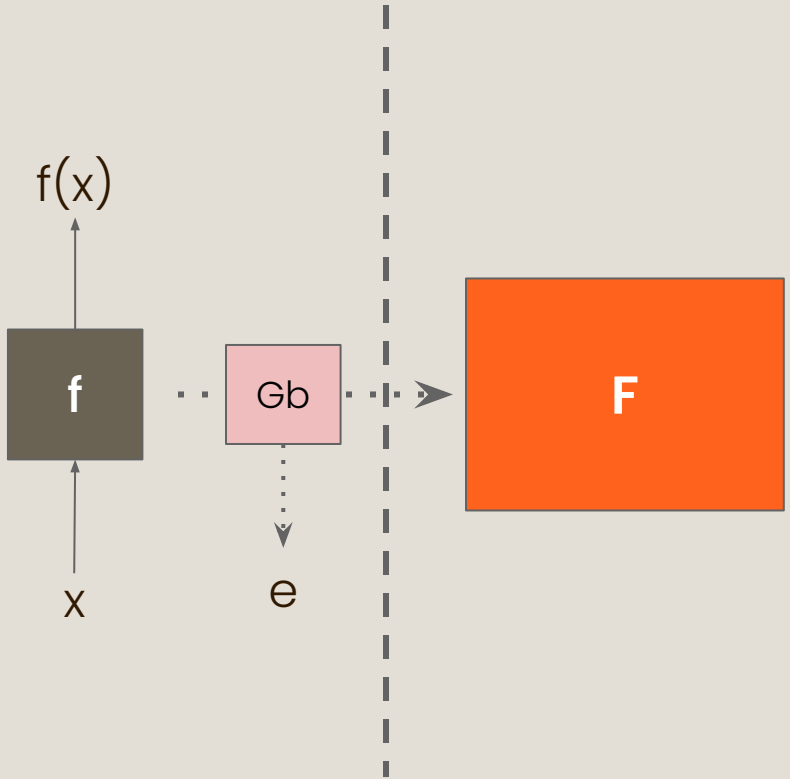

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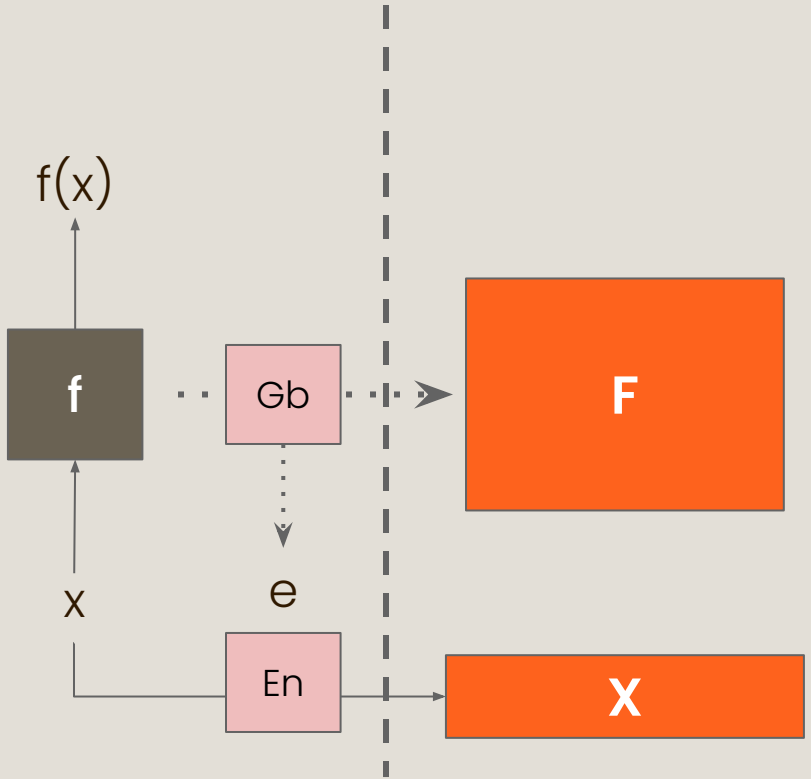

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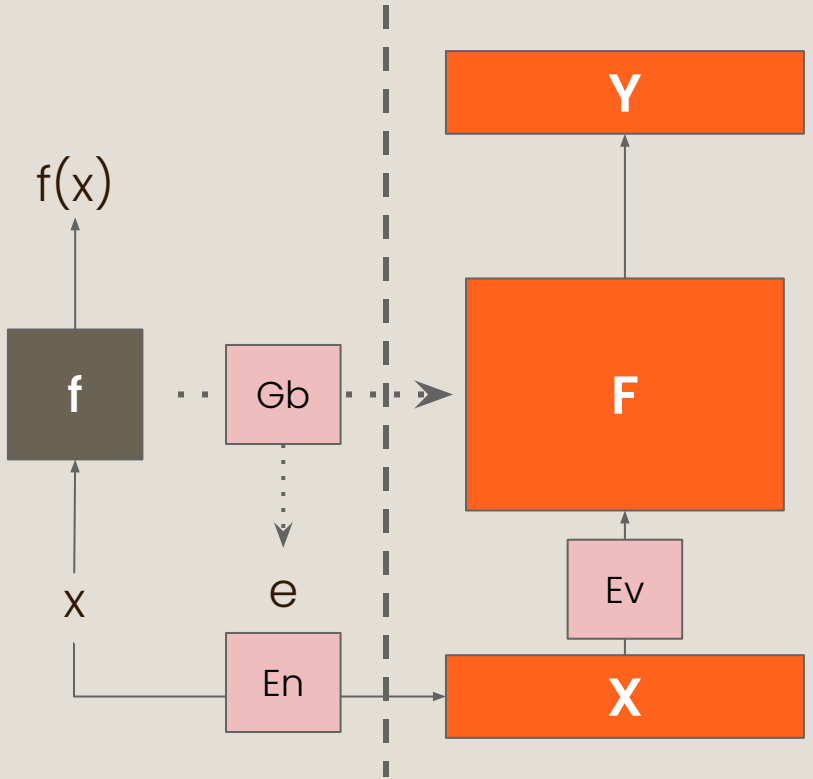

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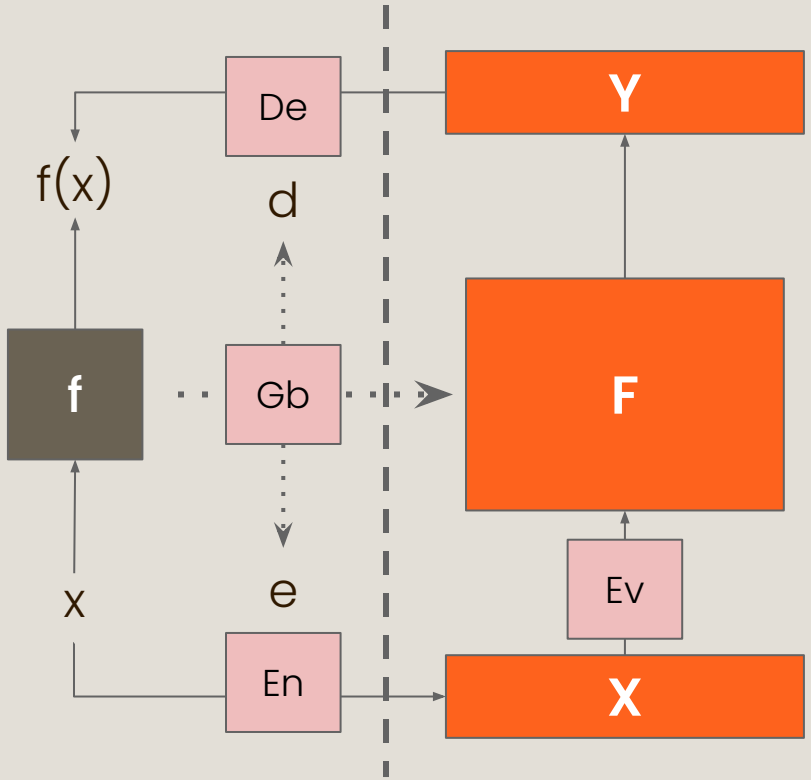

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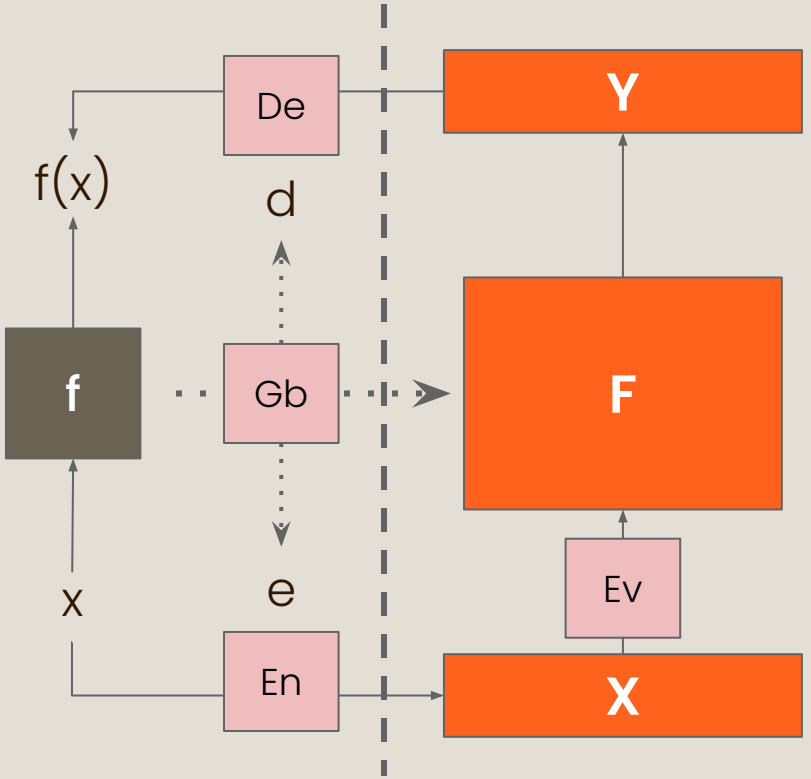
$f \rightarrow C$



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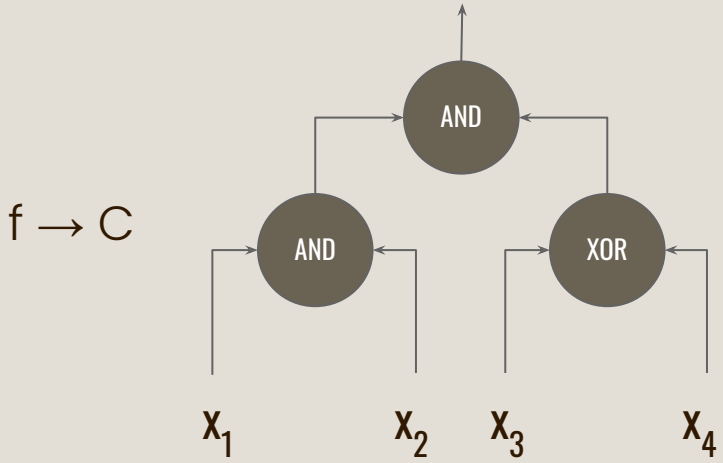

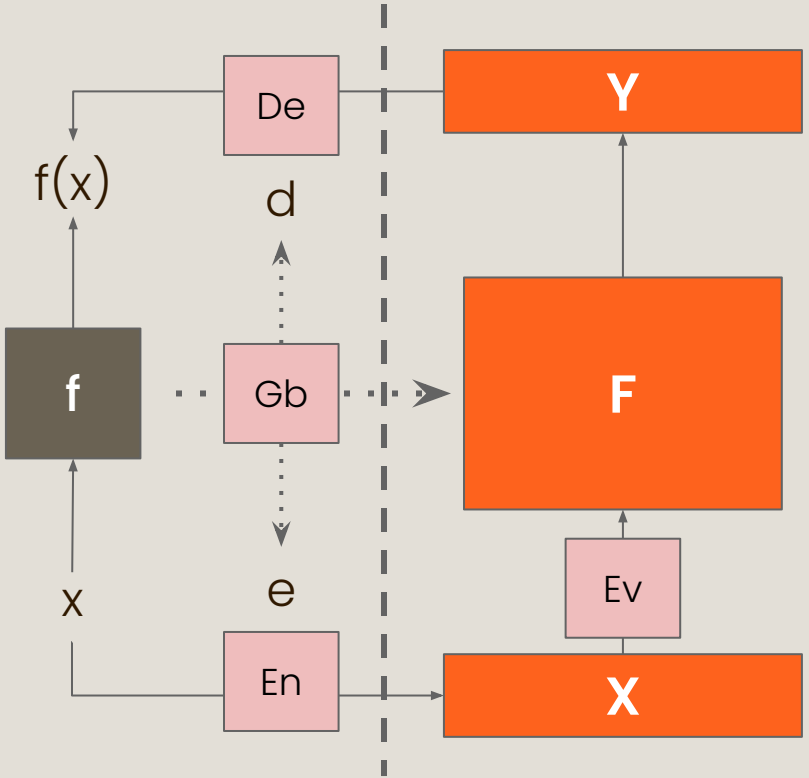

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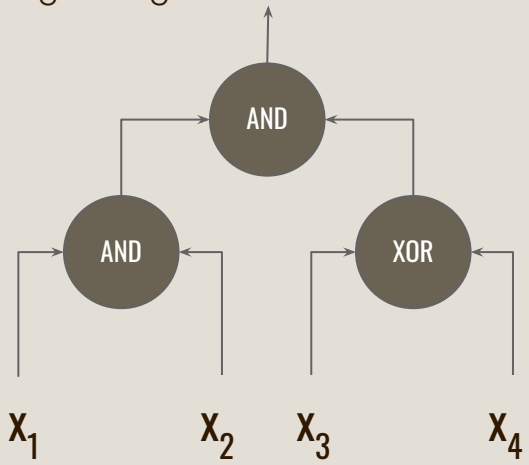
Gate-by-Gate garbling



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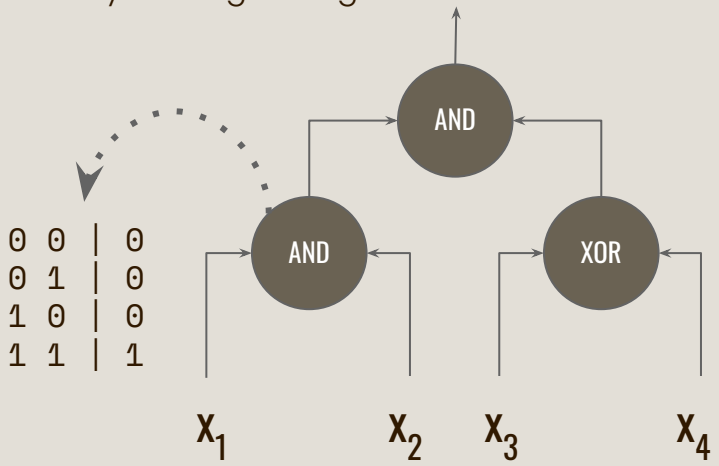

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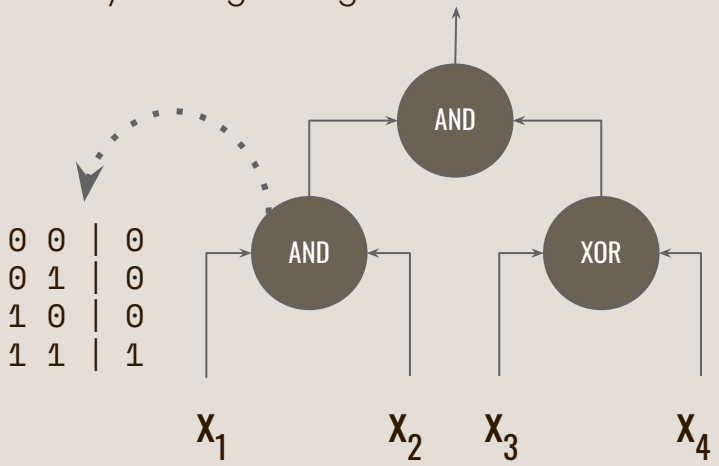

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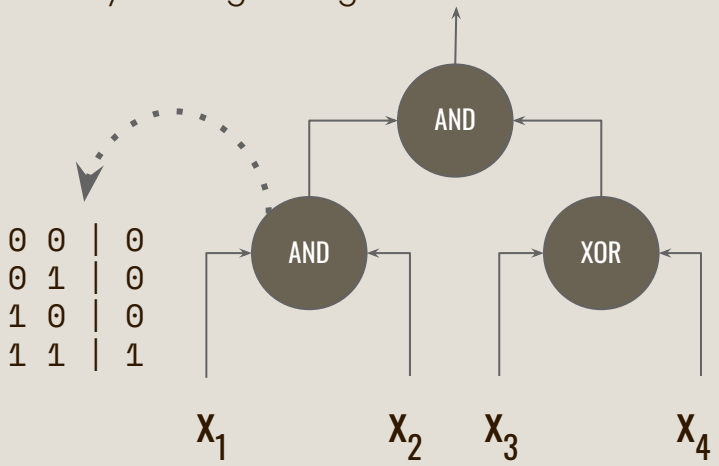

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Free-XOR [KS08]

AND gate: 4K bits

XOR gate: 0 bits

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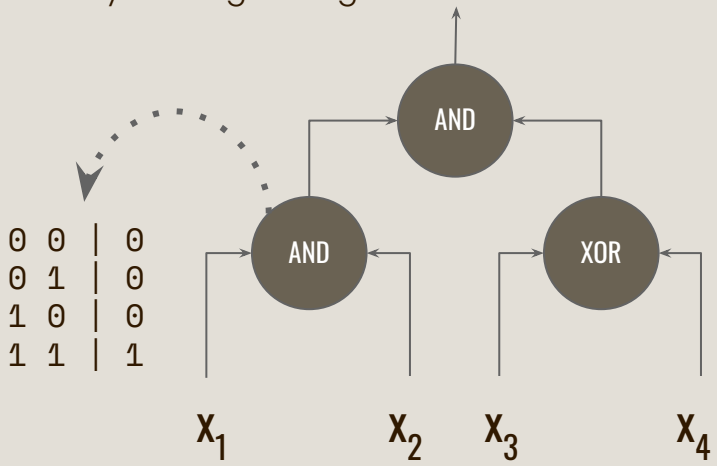

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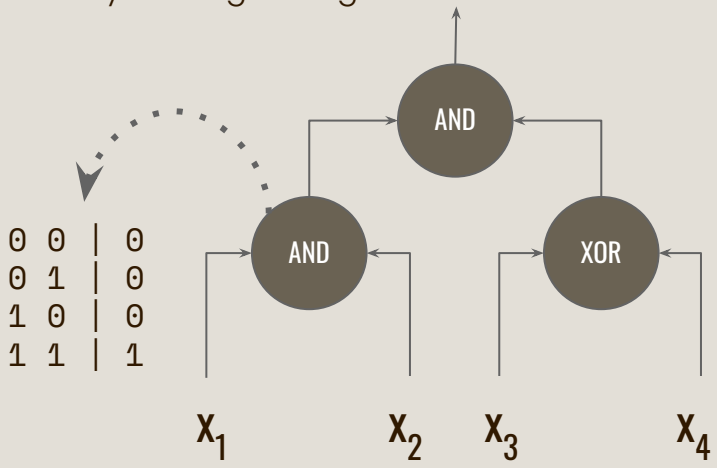

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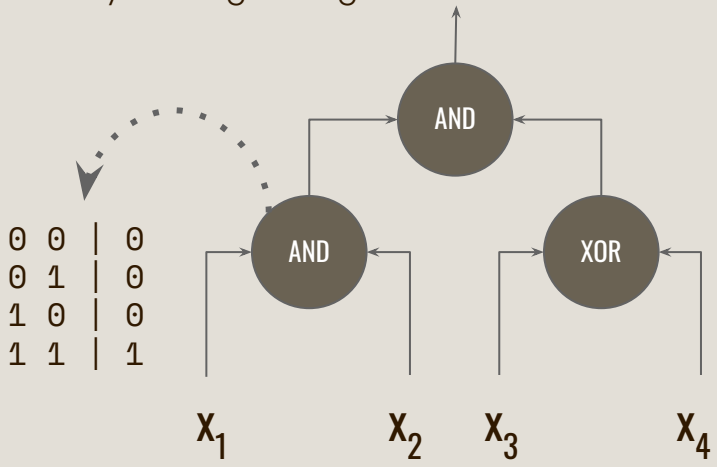

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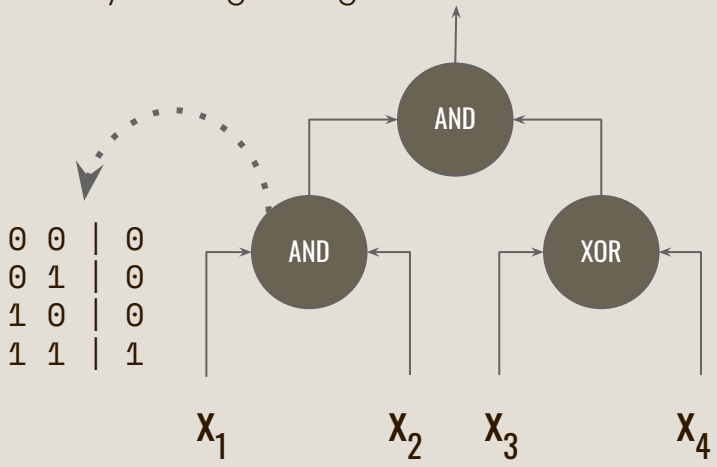

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Requires Circular Security

Circular Correlation Robustness

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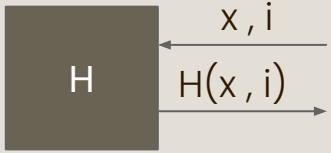

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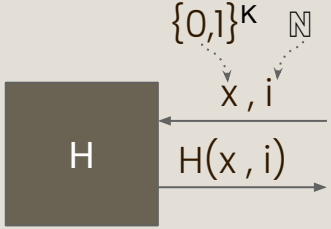

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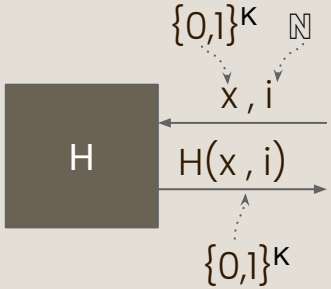

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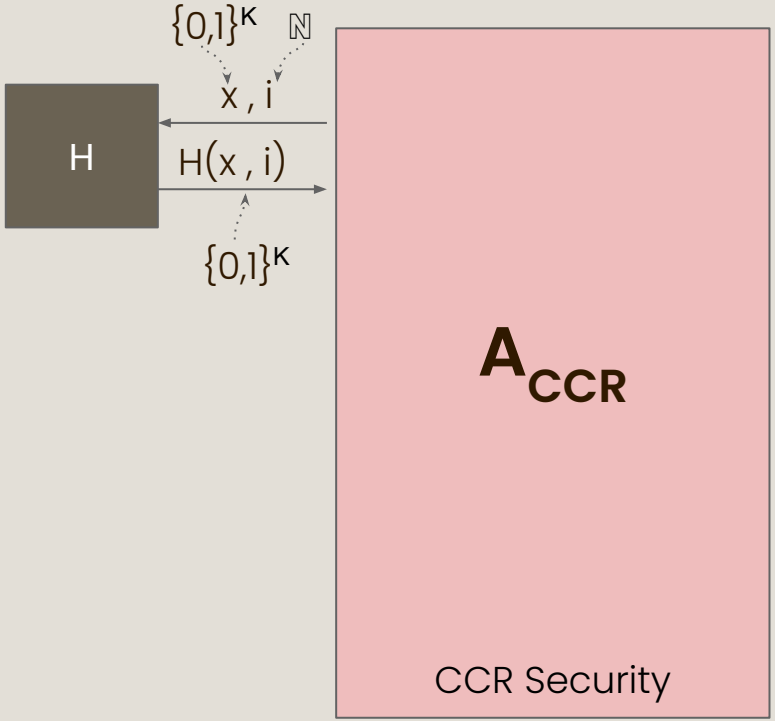

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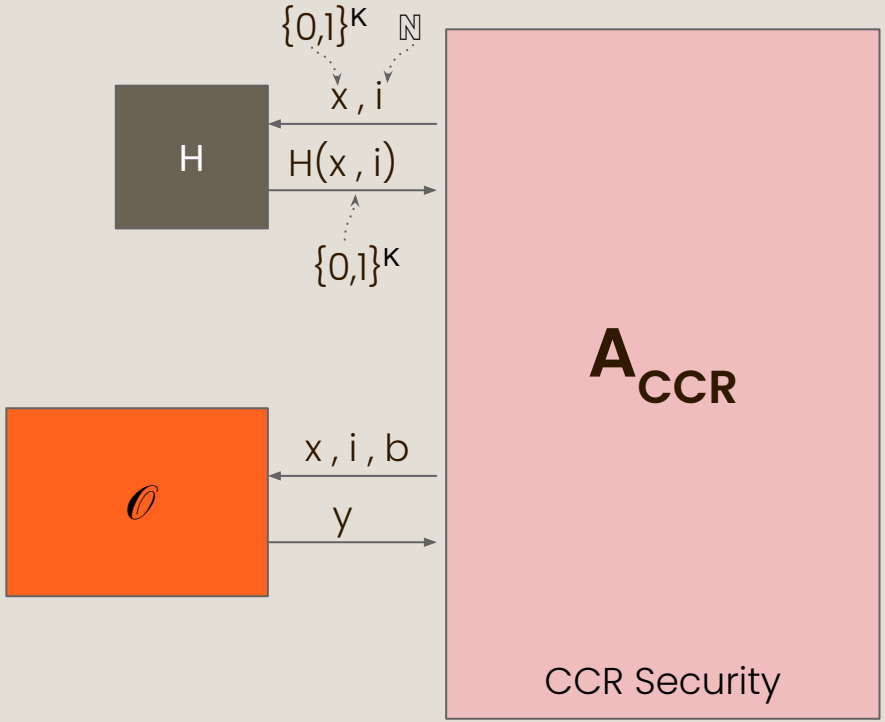

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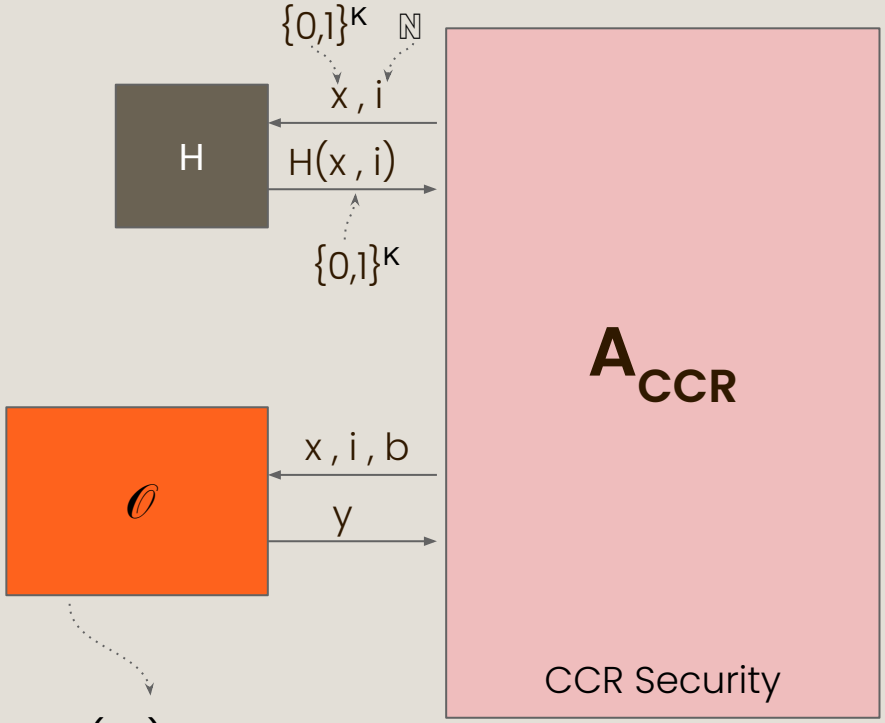

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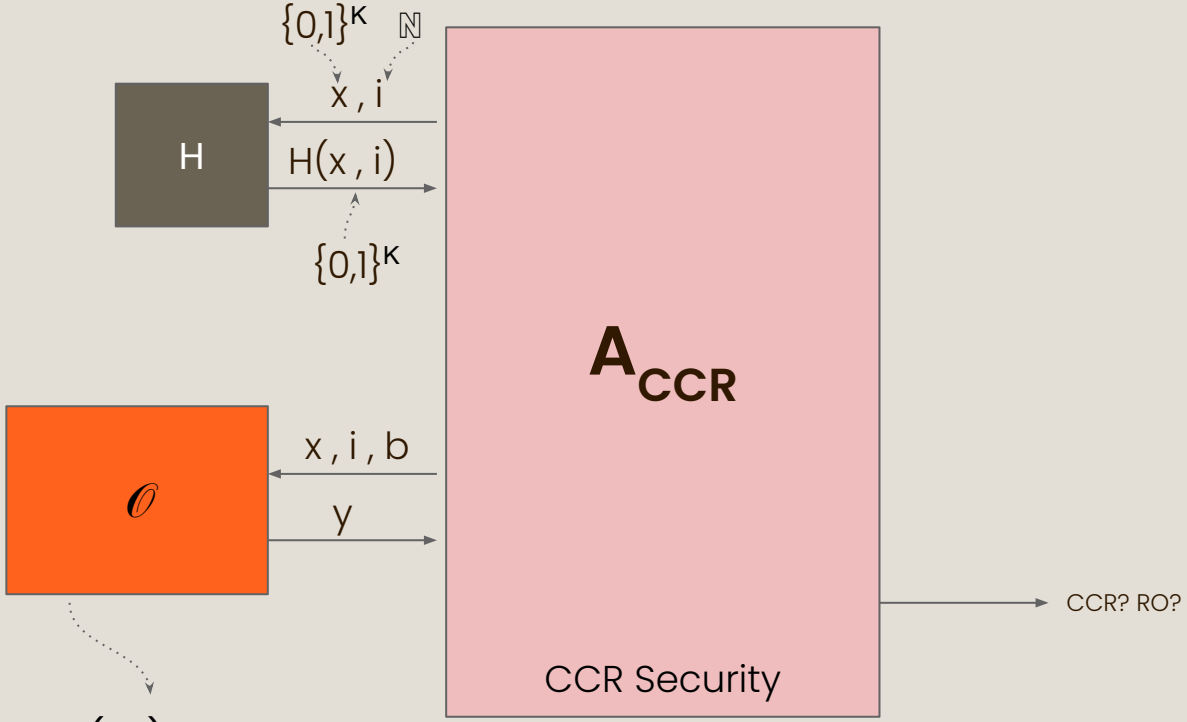

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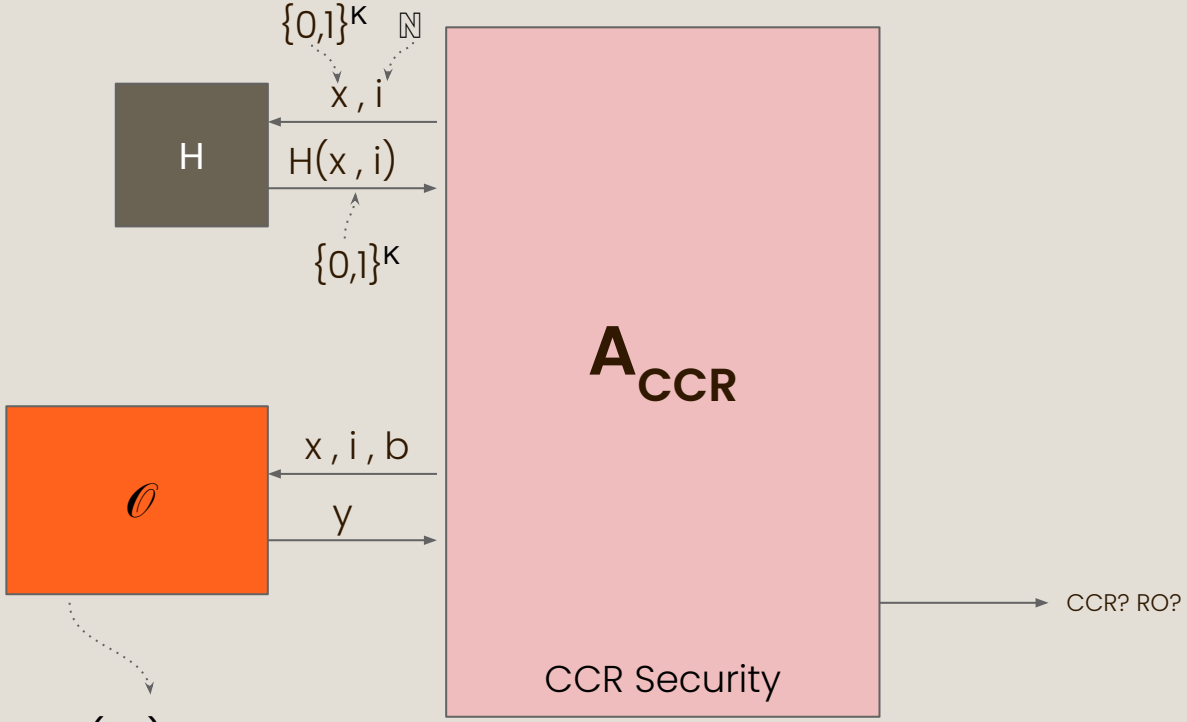

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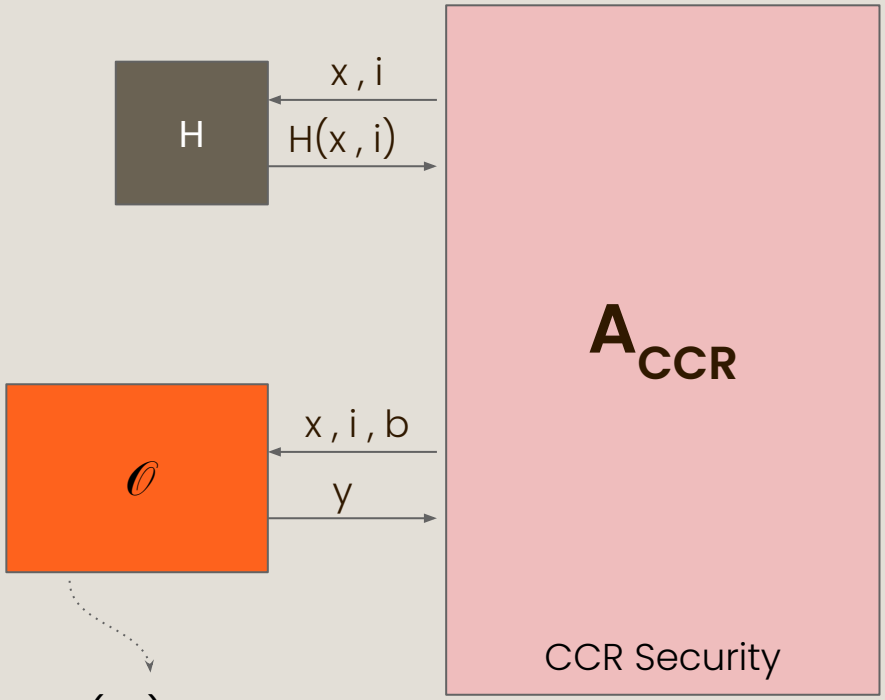

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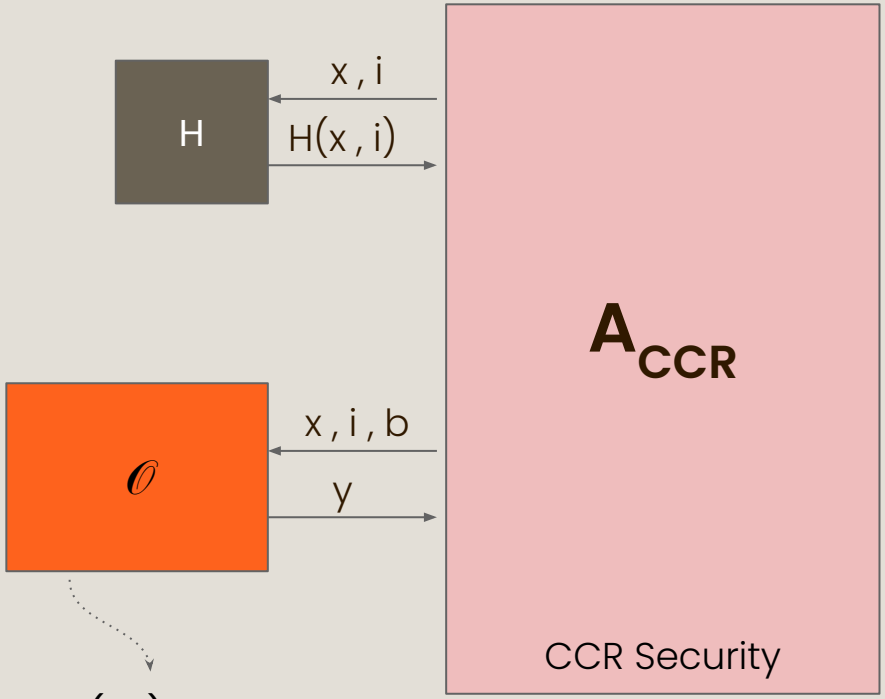

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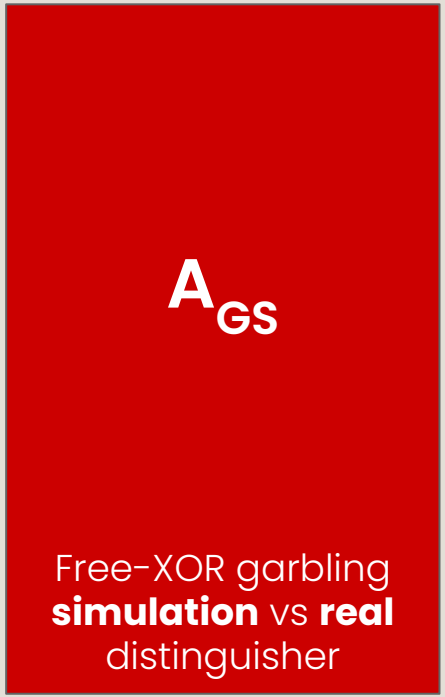

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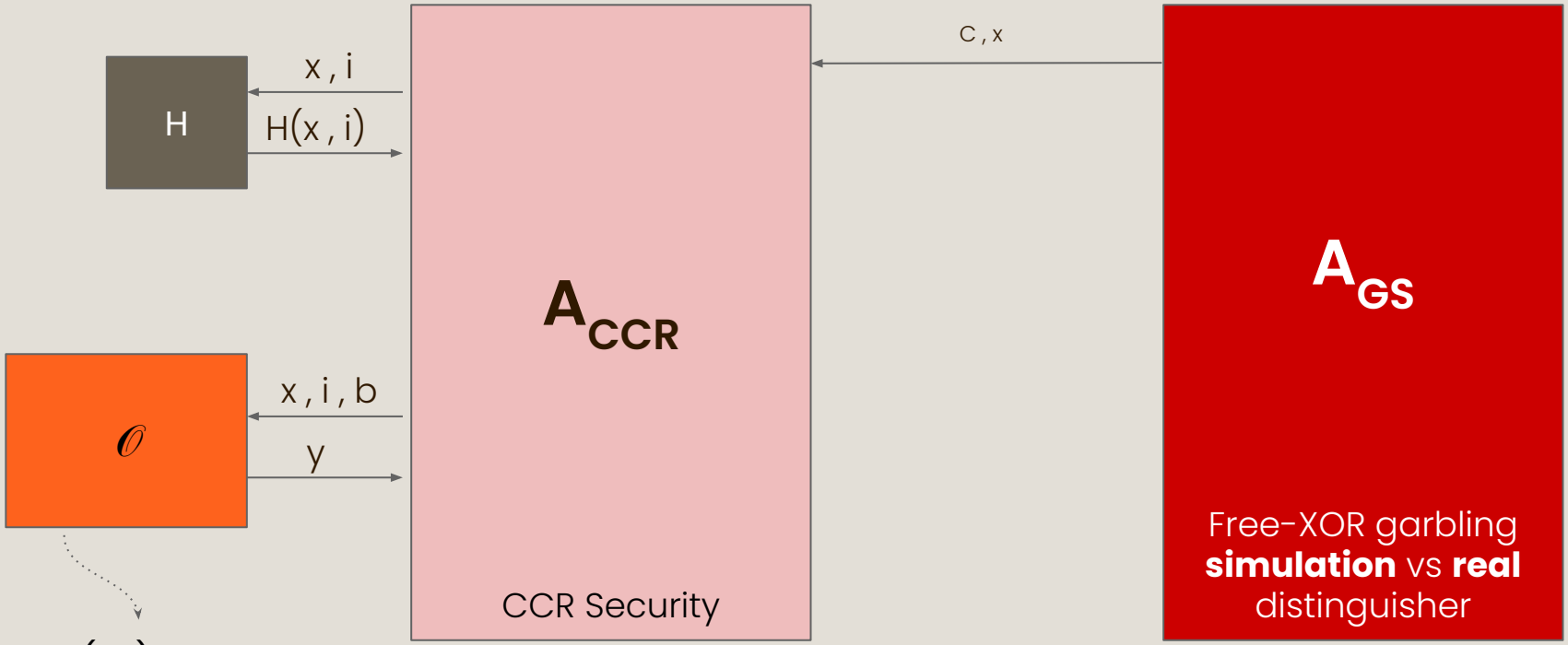

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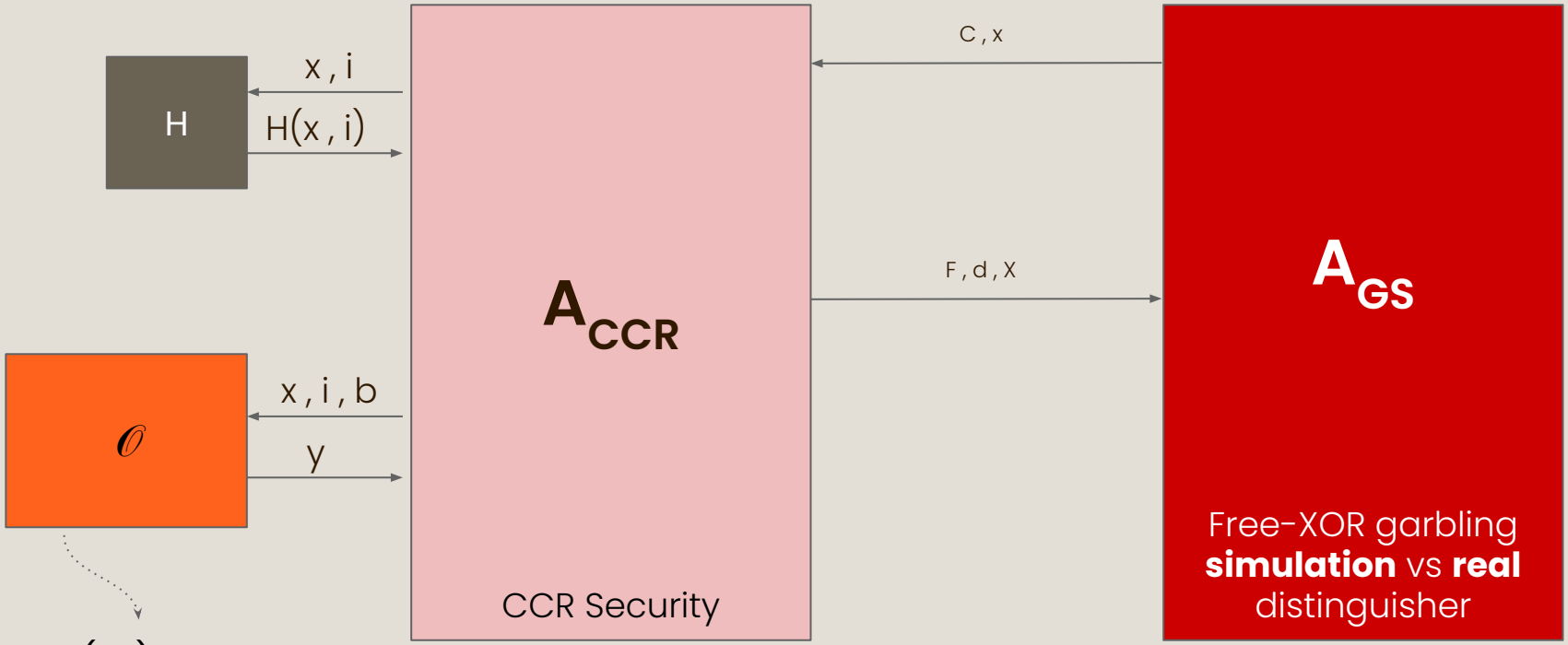

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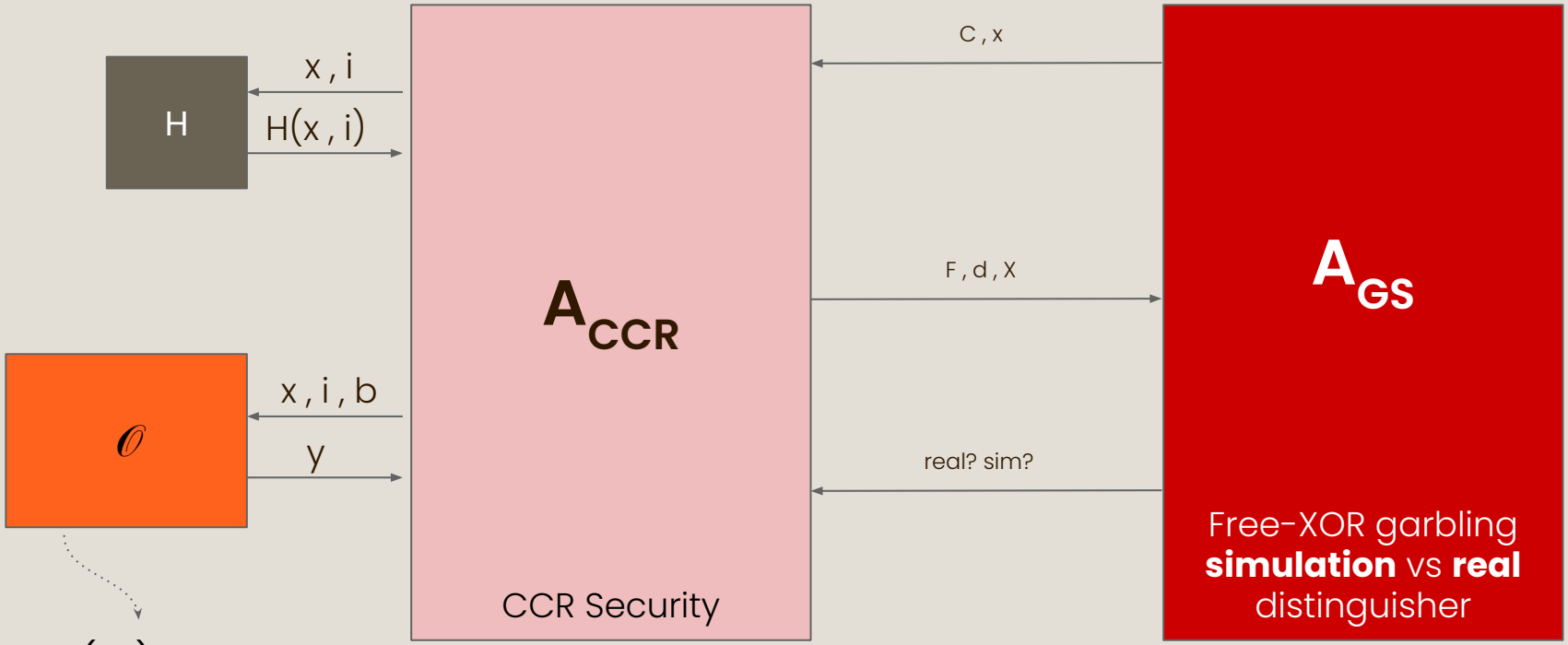

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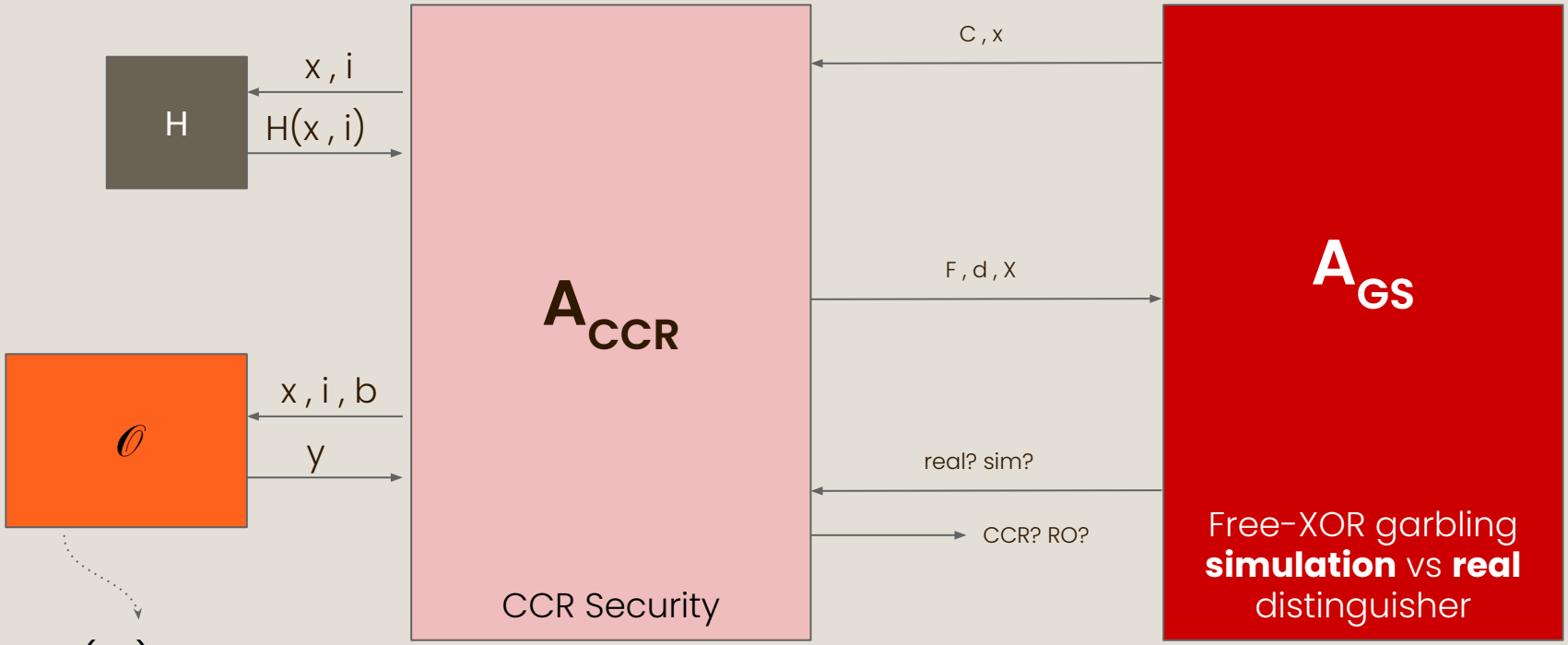

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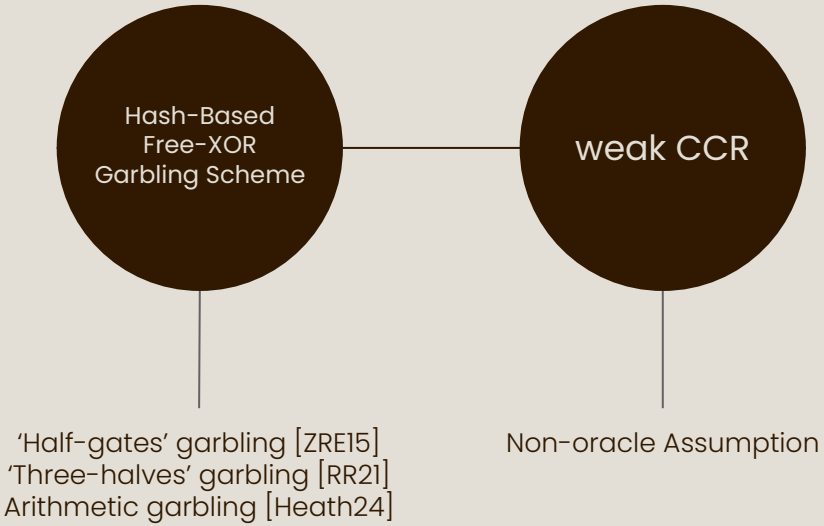

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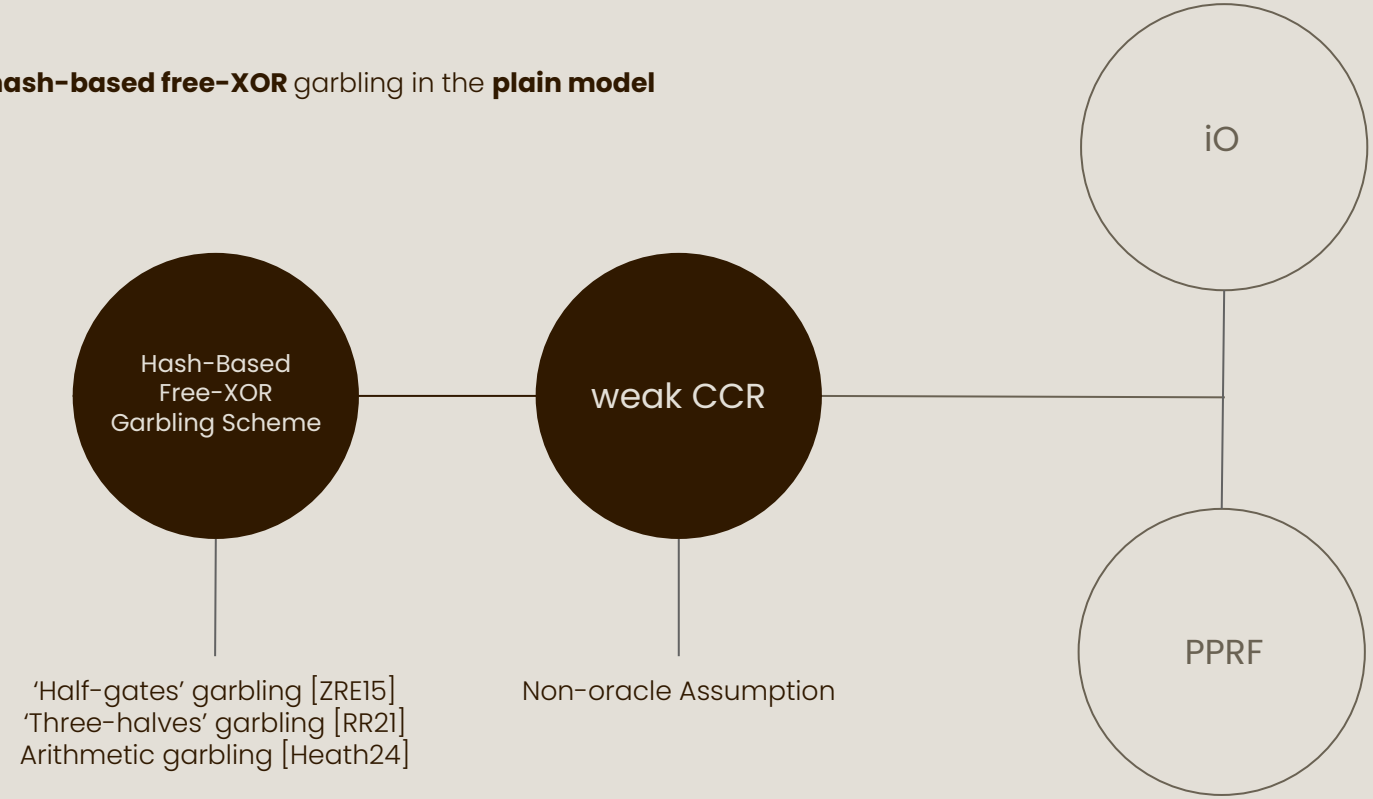

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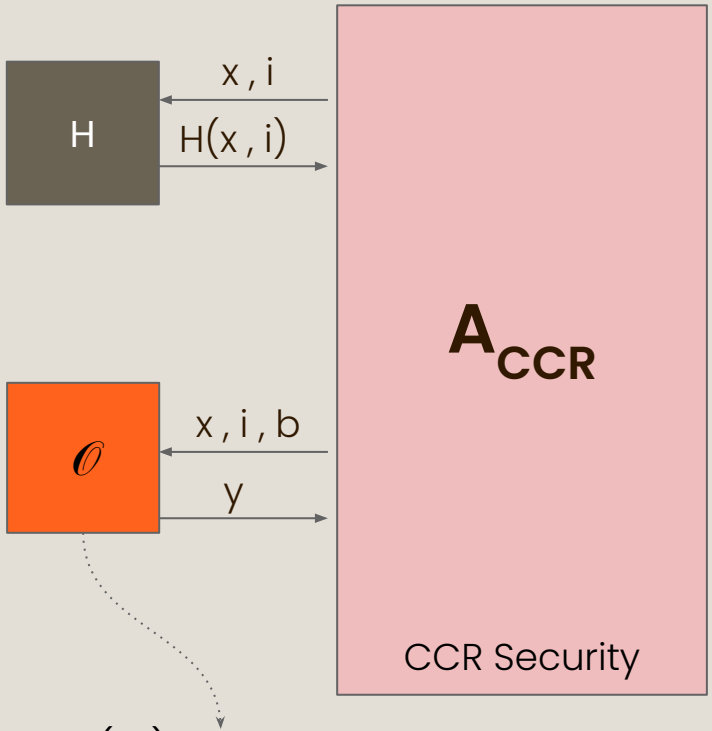

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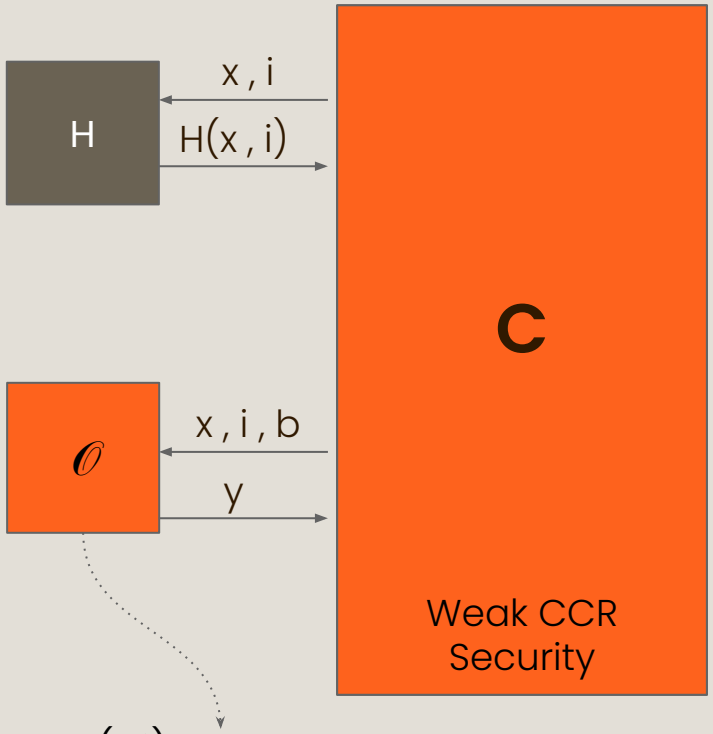

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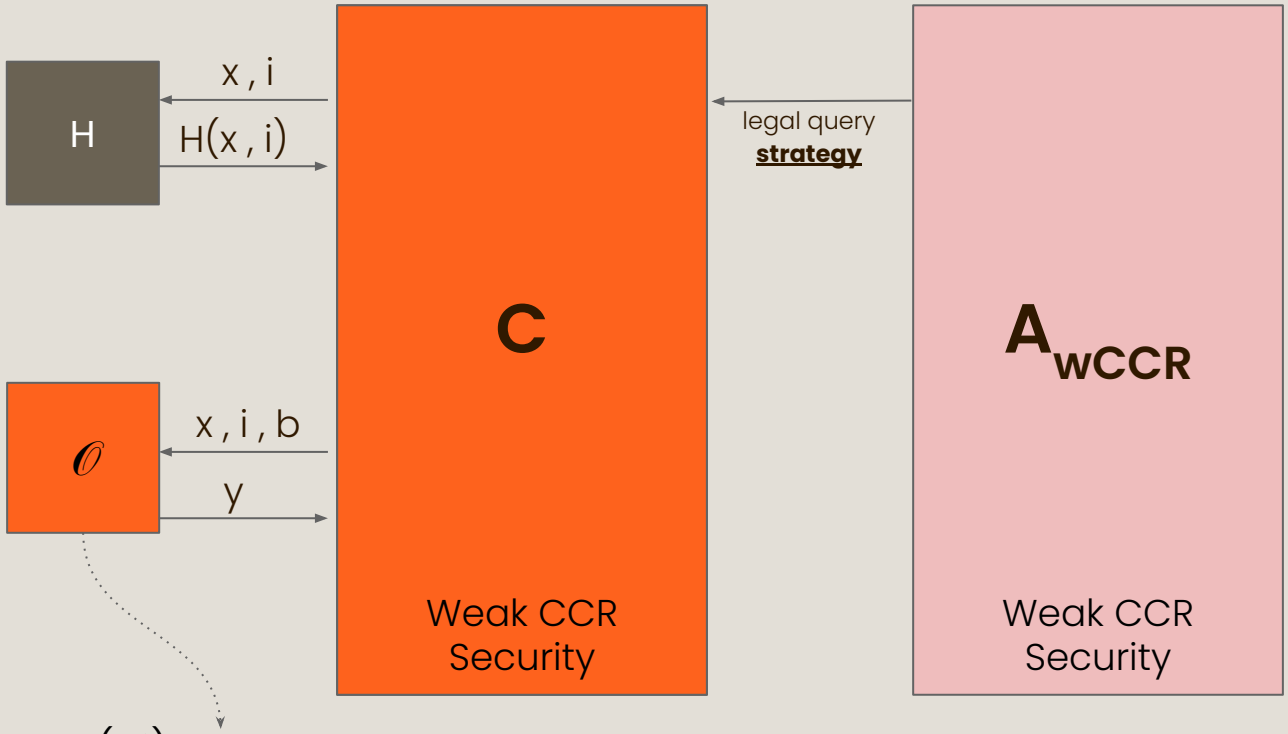

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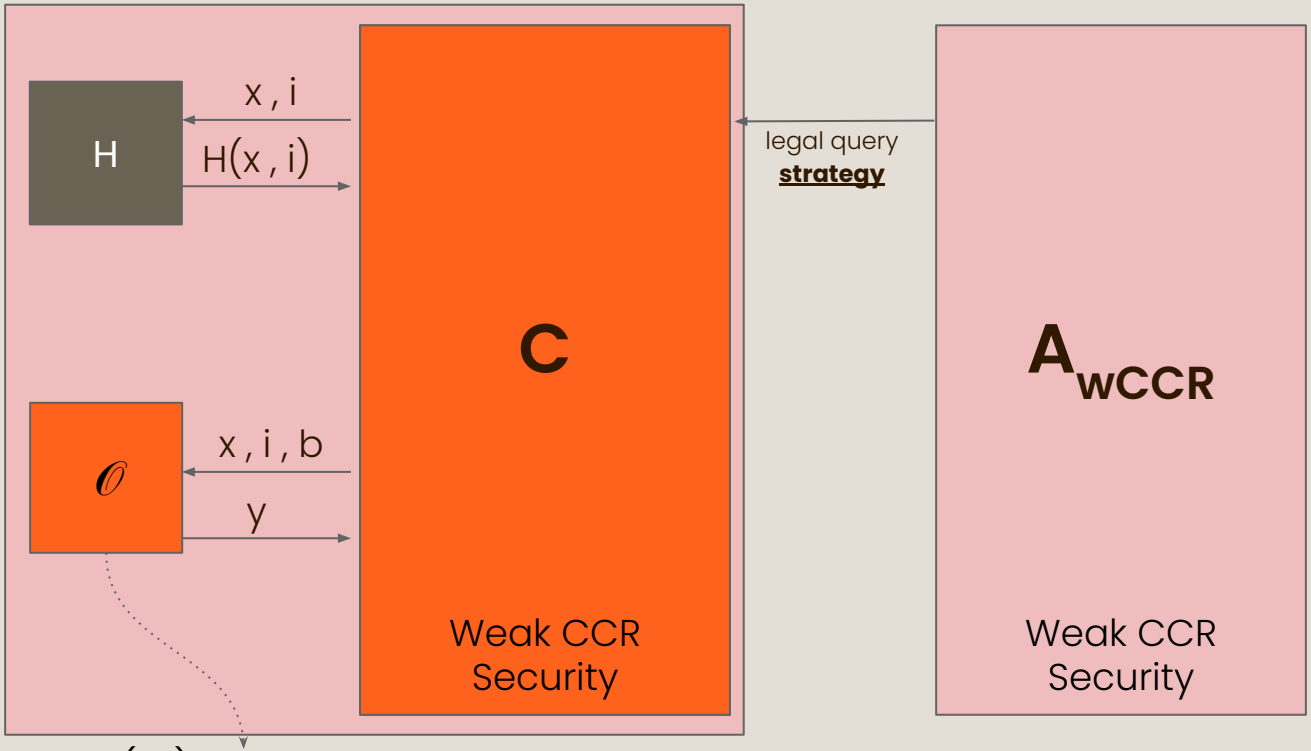

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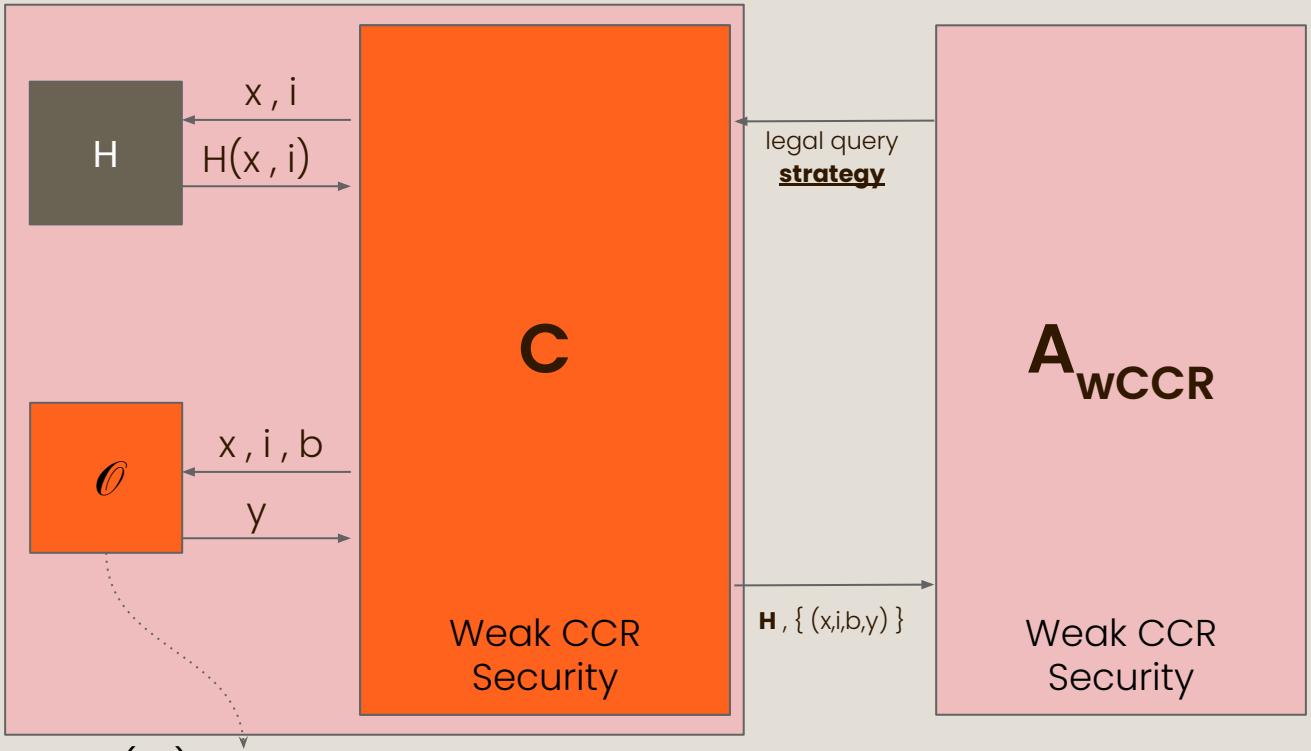

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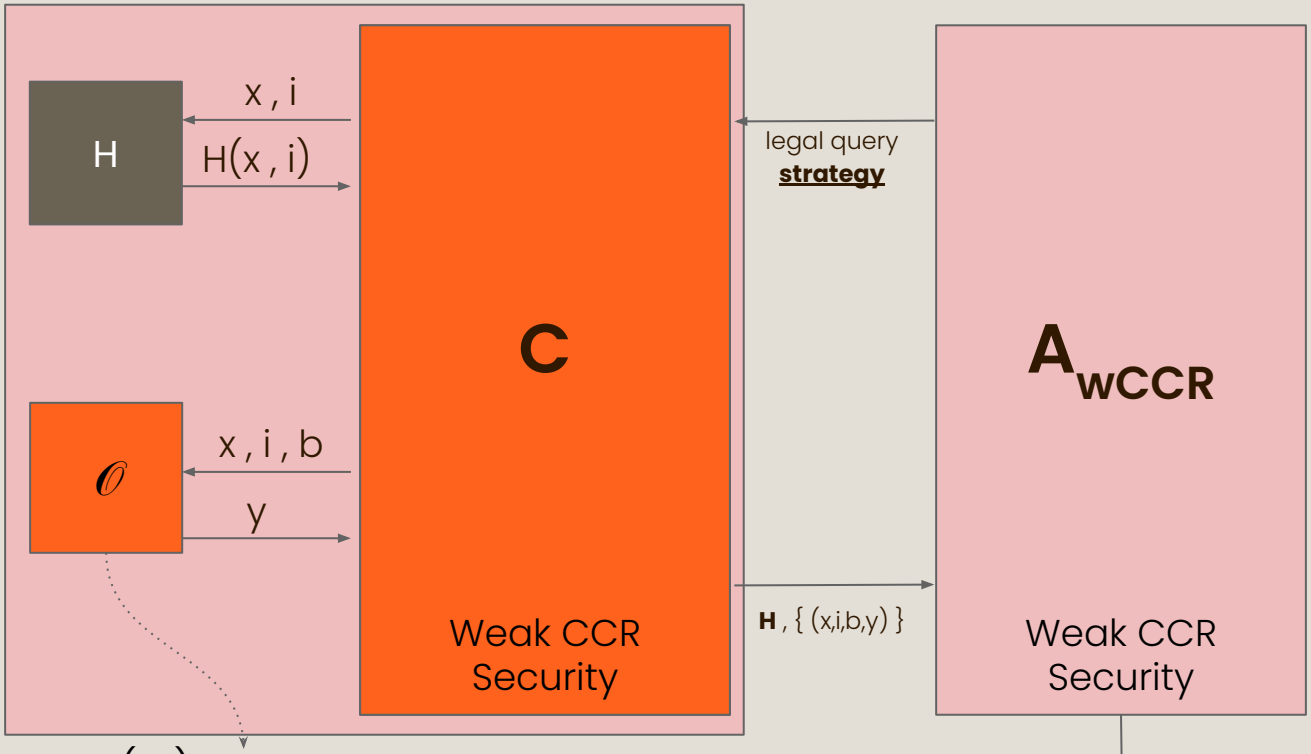

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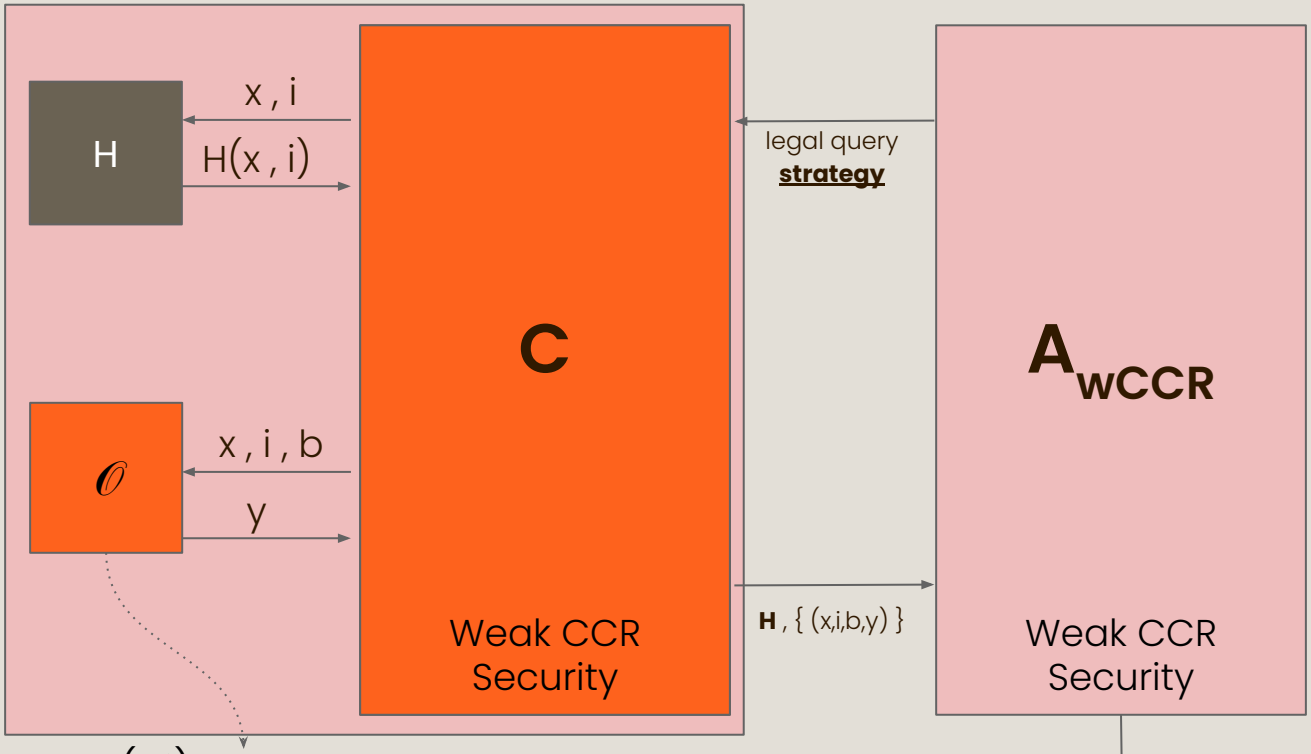

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non-oracle assumption

non-black-box in query strategy

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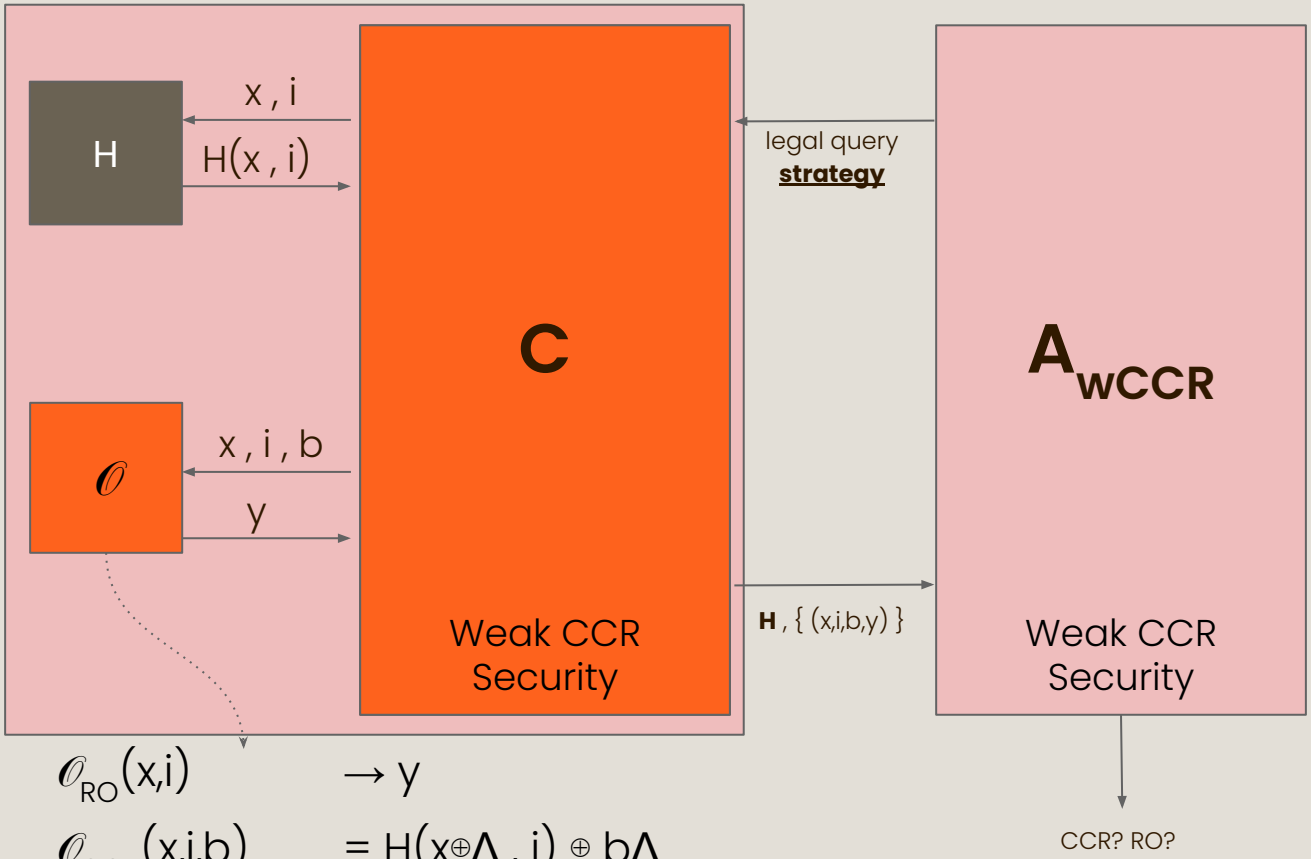

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Difference from RTCCR

|output| = |input|

linear combination defined by b

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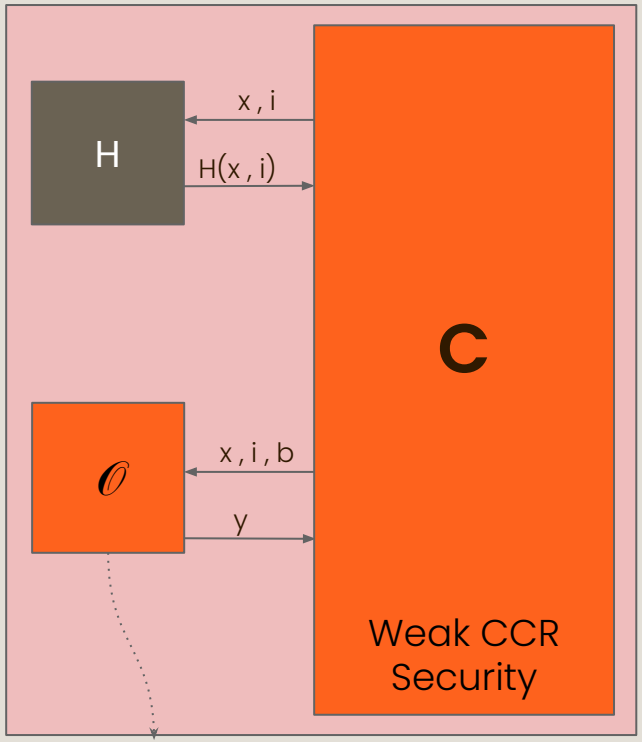

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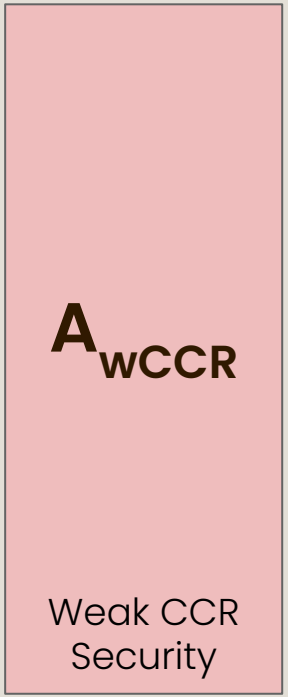
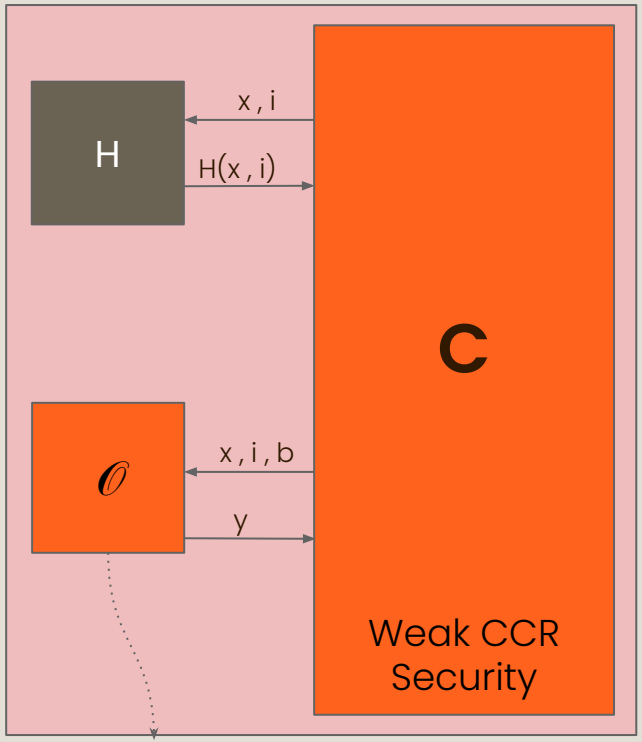


Weak CCR



$\mathcal{O}_{RO}(x,i) \rightarrow y$

$\mathcal{O}_{CCR}(x,i,b) = H(x \oplus \Delta , i) \oplus b\Delta$

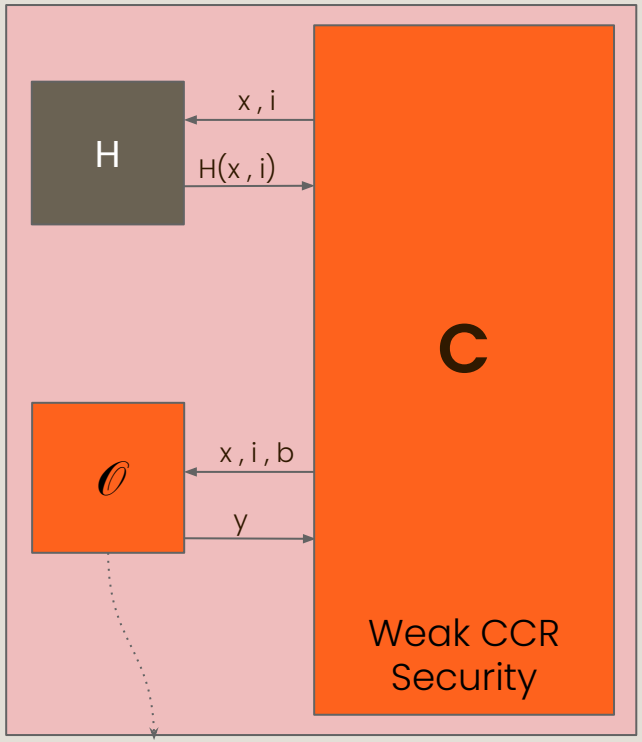
$\mathbf{A}_{wCCR}$

Weak CCR Security

$\mathbf{A}_{GS}$

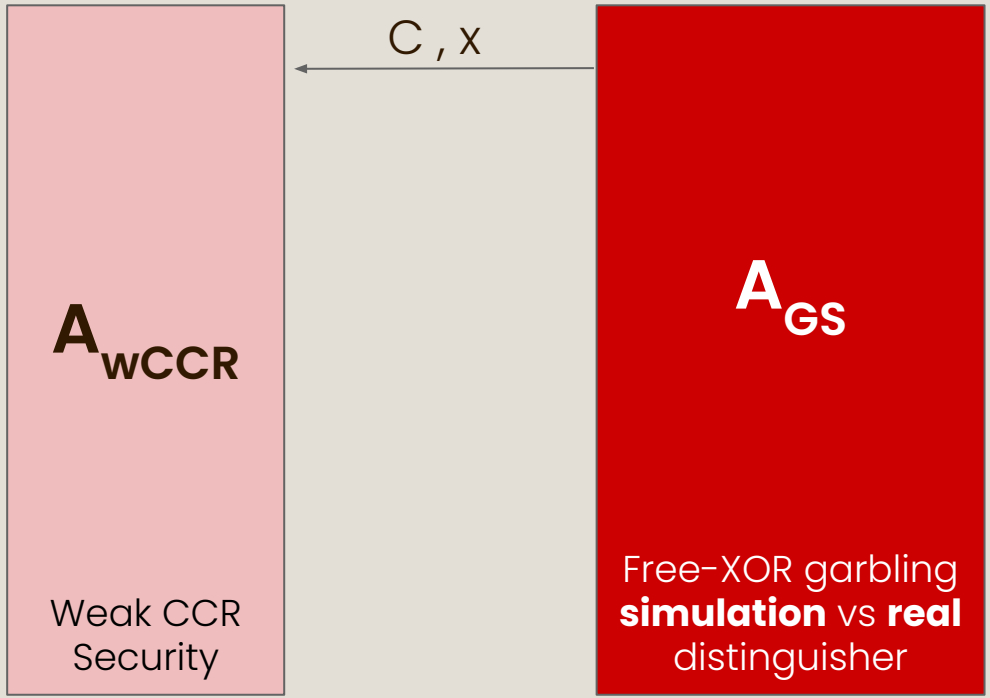
Free-XOR garbling **simulation** vs **real** distinguisher

Weak CCR

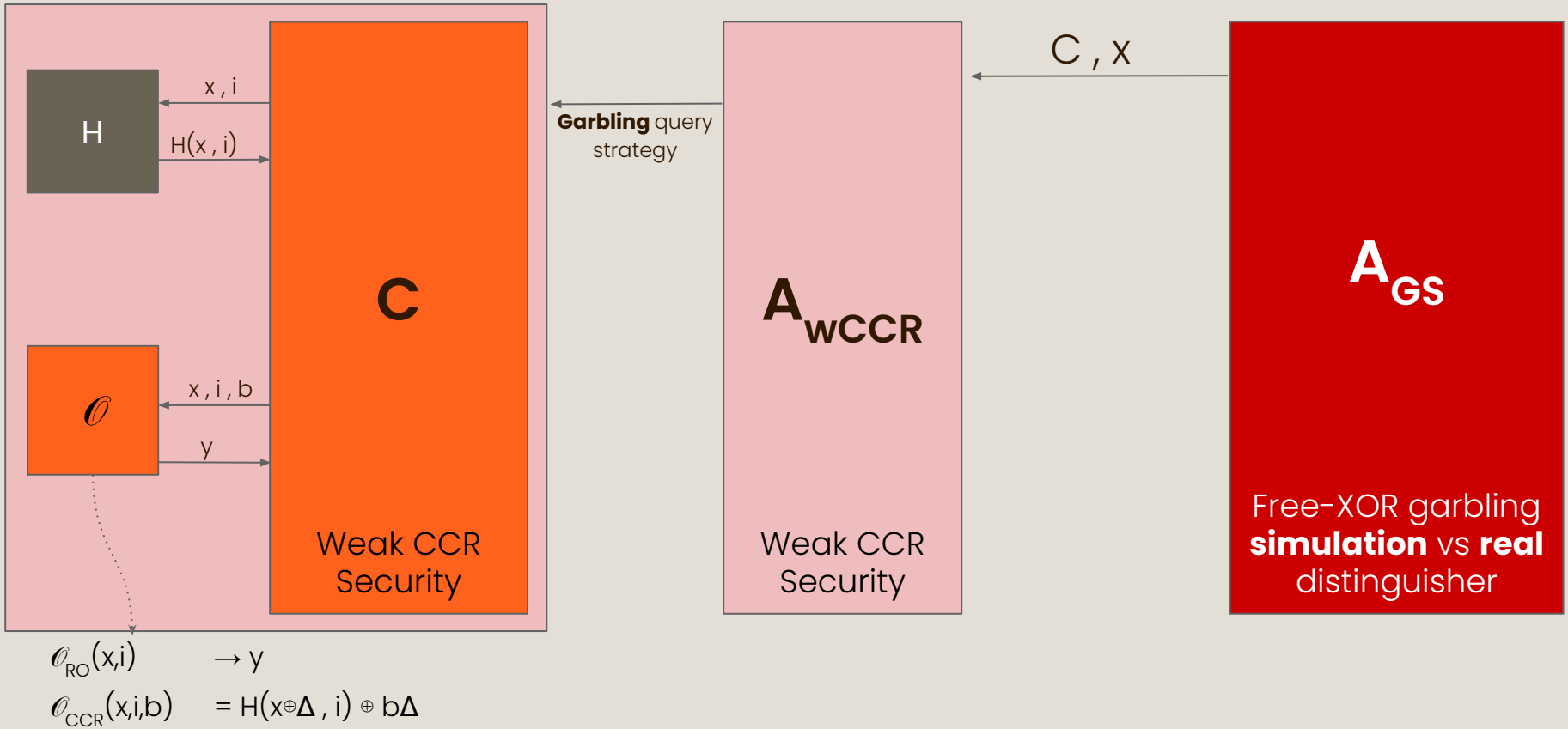


$\mathcal{O}_{RO}(x,i) \rightarrow y$

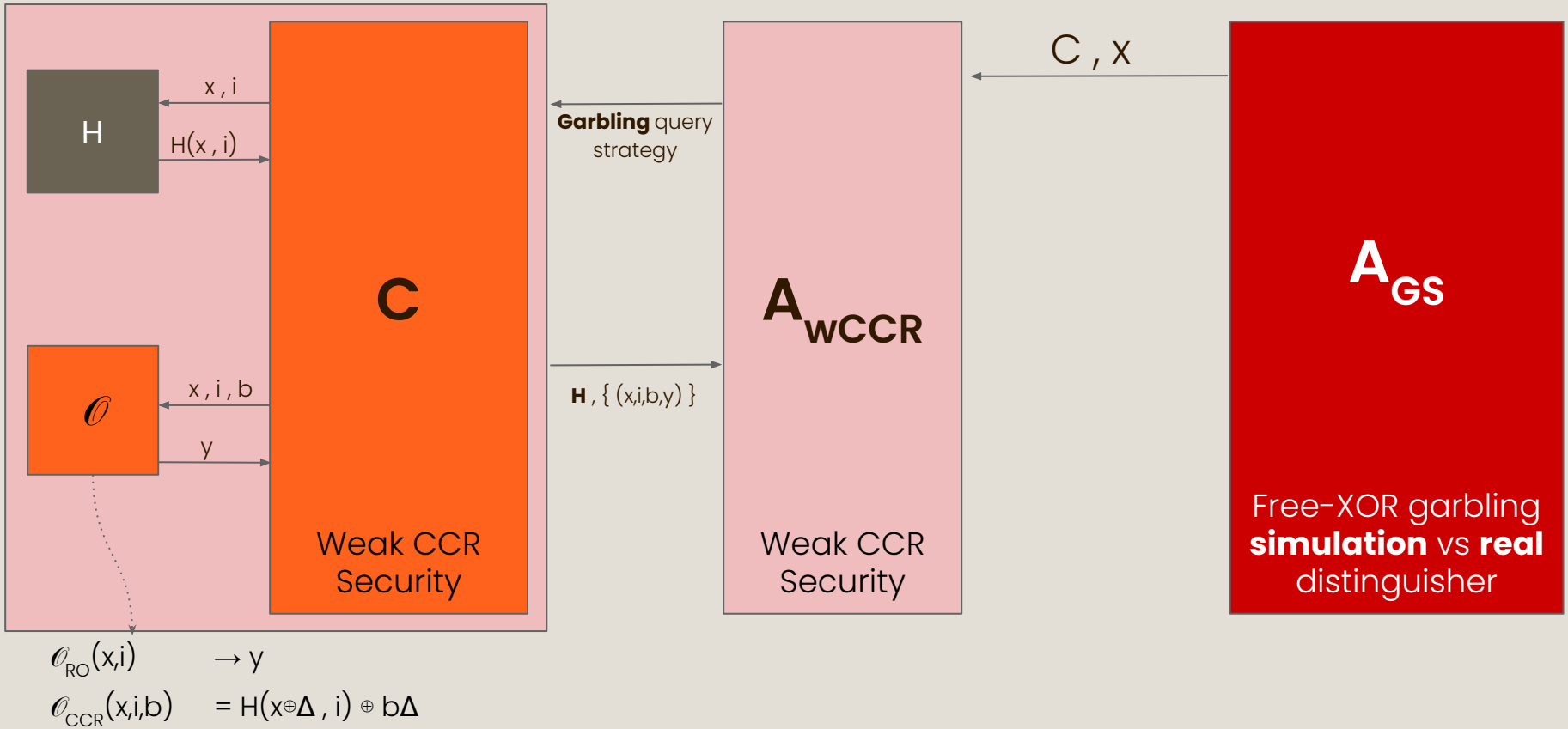
$\mathcal{O}_{CCR}(x,i,b) = H(x \oplus \Delta , i) \oplus b\Delta$



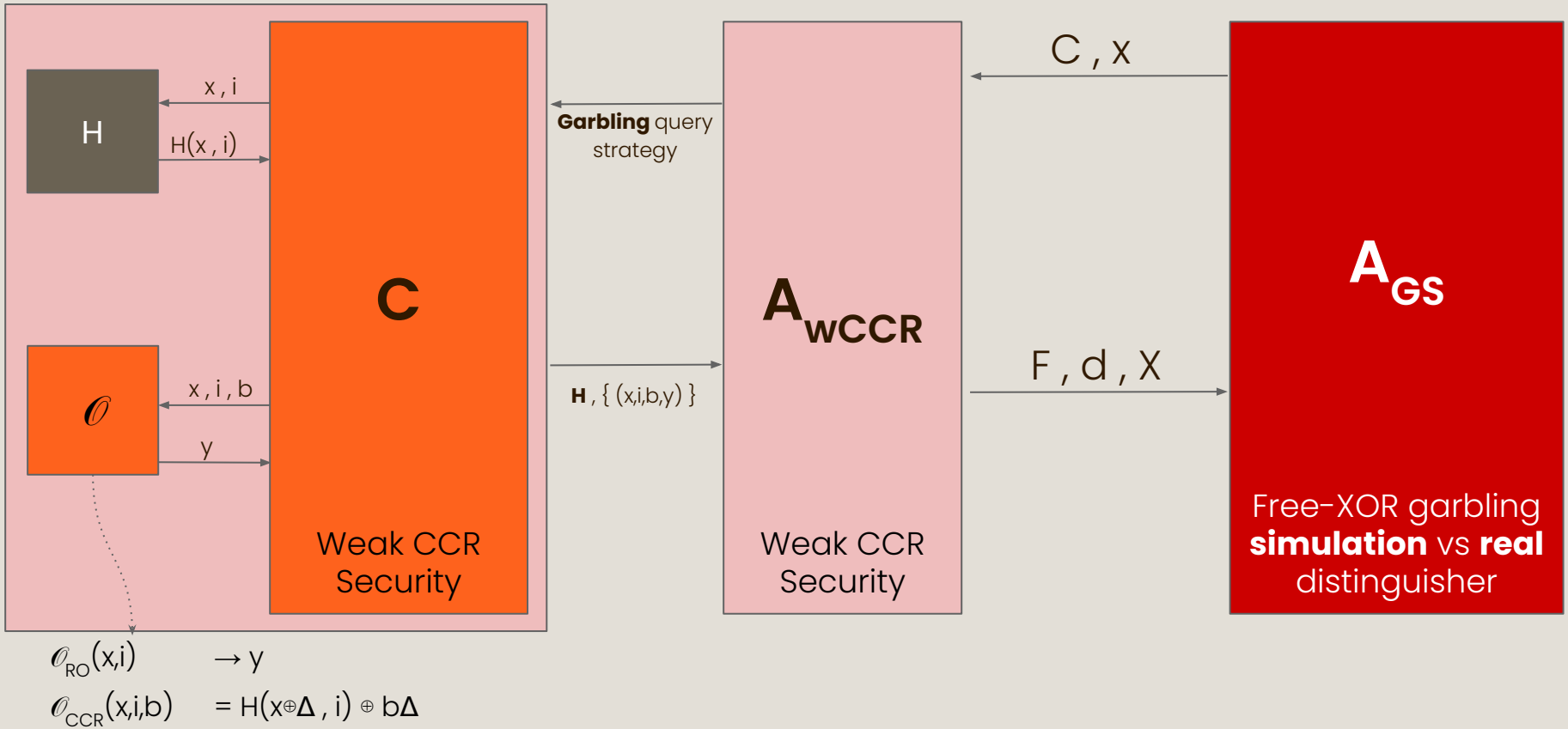
Weak CCR


H
x , i
H(x , i)
C
𝒪
x , i , b
y
Weak CCR Security
Garbling query strategy
A_wCCR
Weak CCR Security
C , x
A_GS
Free-XOR garbling **simulation** vs **real** distinguisher
$\mathcal{O}_{RO}(x,i) \rightarrow y$
$\mathcal{O}_{CCR}(x,i,b) = H(x \oplus \Delta , i) \oplus b\Delta$

Weak CCR


H
x , i
H(x , i)
C
𝒪
x , i , b
y
Weak CCR Security
Garbling query strategy
H , { (x,i,b,y) }
A wCCR
Weak CCR Security
C , x
A GS
Free-XOR garbling simulation vs real distinguisher
𝒪RO(x,i) → y
𝒪CCR(x,i,b) = H(x⊕Δ , i) ⊕ bΔ

Weak CCR


H
x , i
H(x , i)
C
𝒪
x , i , b
y
Weak CCR Security
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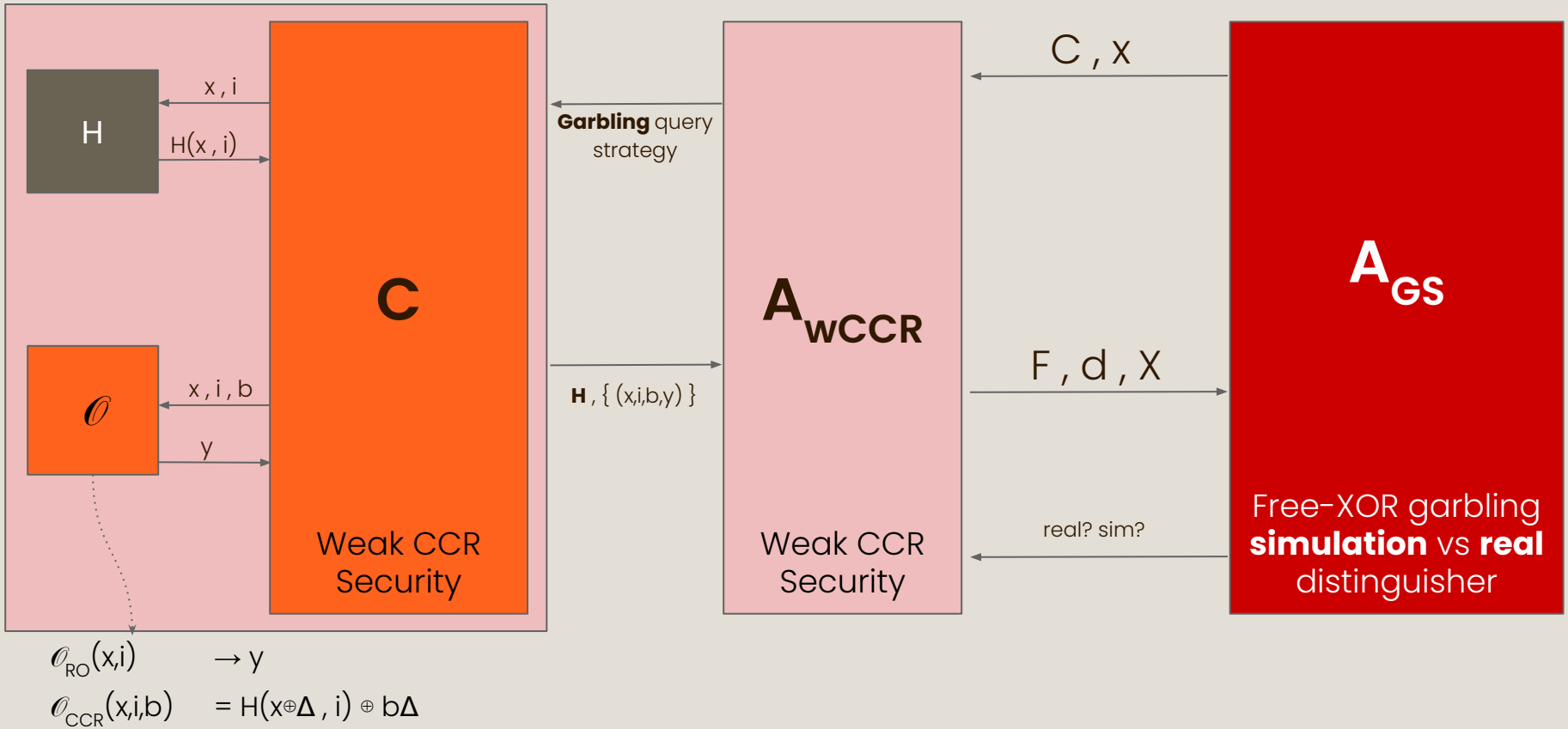
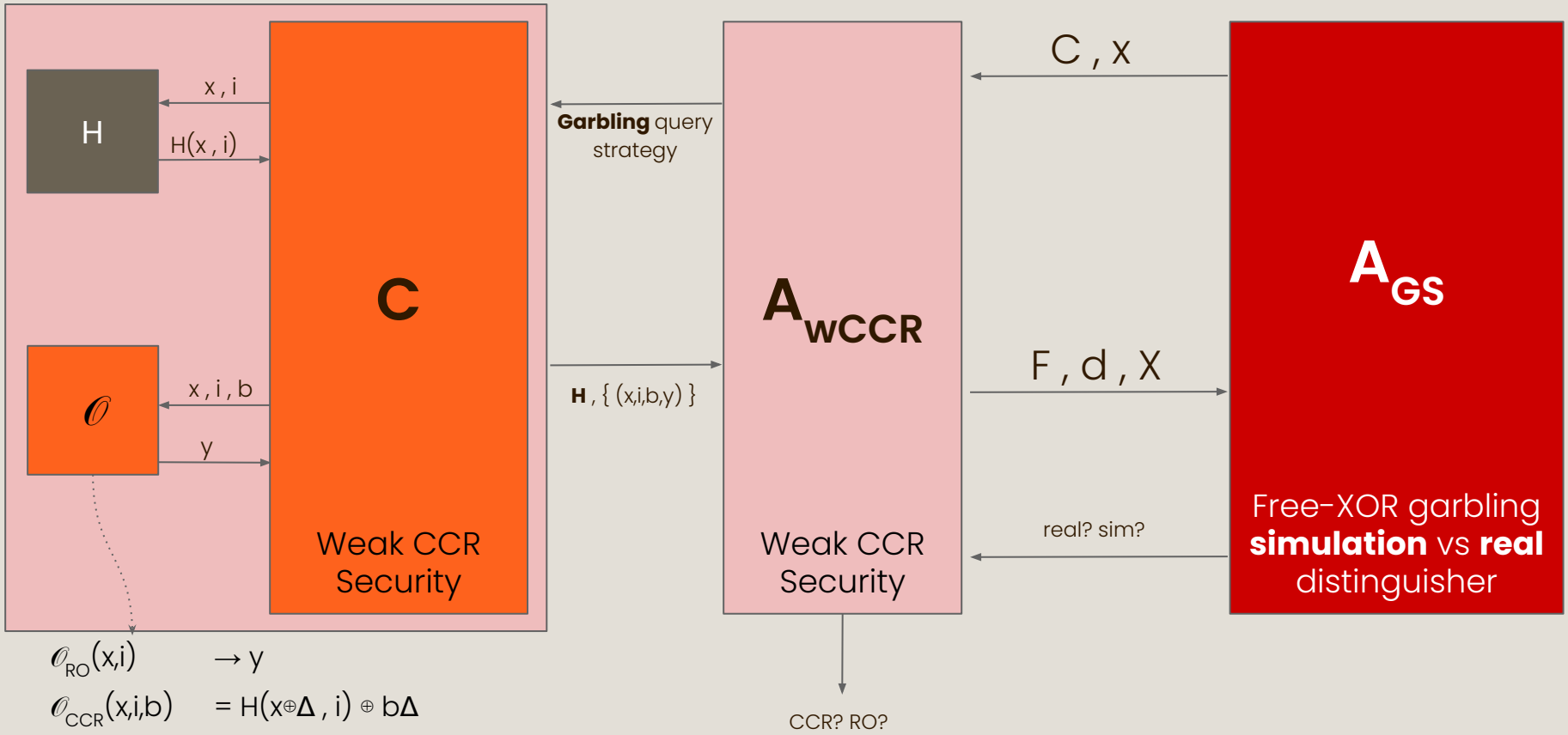
H , { (x,i,b,y) }
A wCCR
Weak CCR Security
C , x
F , d , X
A GS
Free-XOR garbling simulation vs real distinguisher
𝒪RO(x,i) → y
𝒪CCR(x,i,b) = H(x⊕Δ , i) ⊕ bΔ

Weak CCR


H
x , i
H(x , i)
C
𝒪
x , i , b
y
Weak CCR Security
Garbling query strategy
H , { (x,i,b,y) }
A_wCCR
Weak CCR Security
C , x
F , d , X
real? sim?
A_GS
Free-XOR garbling simulation vs real distinguisher
𝒪_RO(x,i) → y
𝒪_CCR(x,i,b) = H(x⊕Δ , i) ⊕ bΔ

Weak CCR


H
x , i
H(x , i)
C
$\mathcal{O}$
x , i , b
y
Weak CCR Security
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$\mathcal{O}_{CCR}$(x,i,b) = H(x⊕Δ , i) ⊕ bΔ
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H , { (x,i,b,y) }
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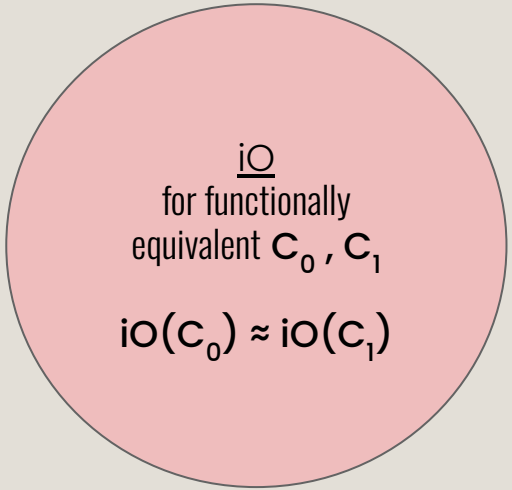
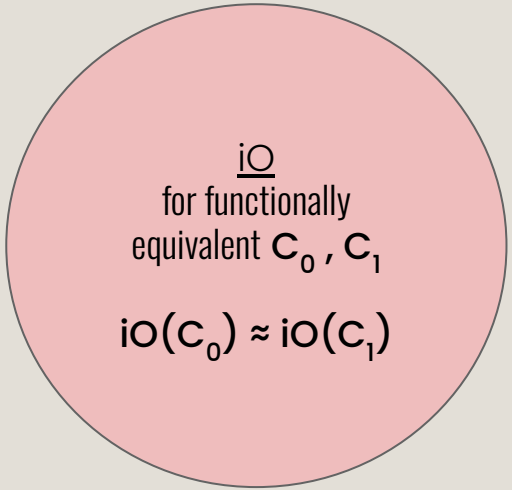
Weak CCR Security
CCR? RO?
C , x
F , d , X
real? sim?
A_{GS}
Free-XOR garbling
simulation vs real
distinguisher

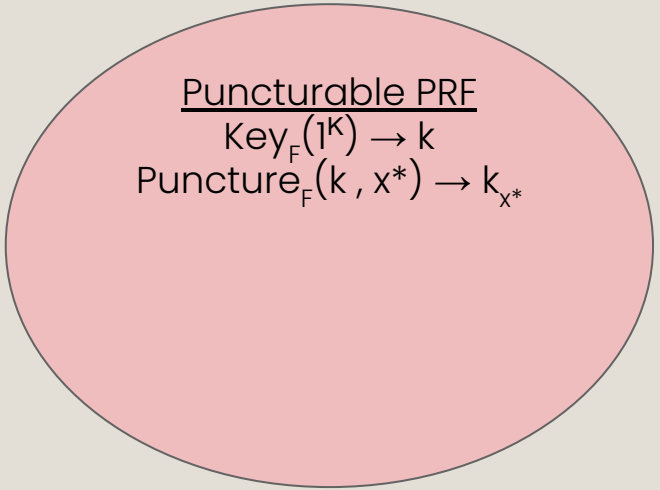
CONSTRUCTION

CONSTRUCTION

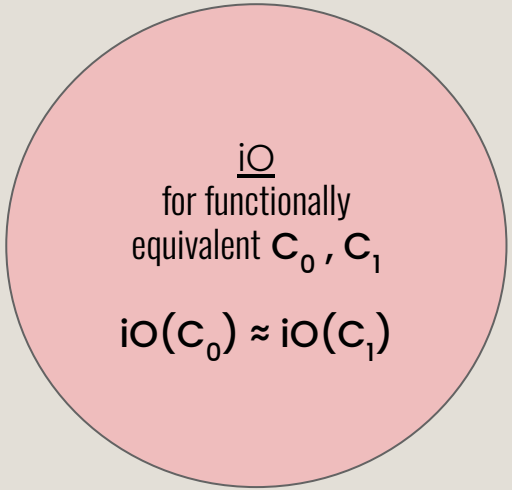


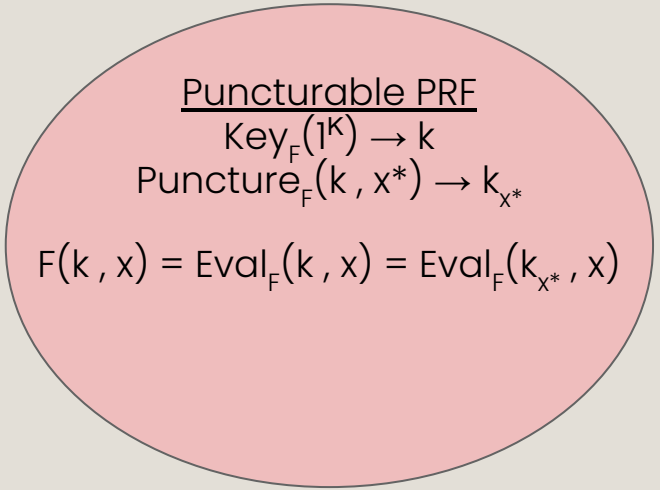
CONSTRUCTION



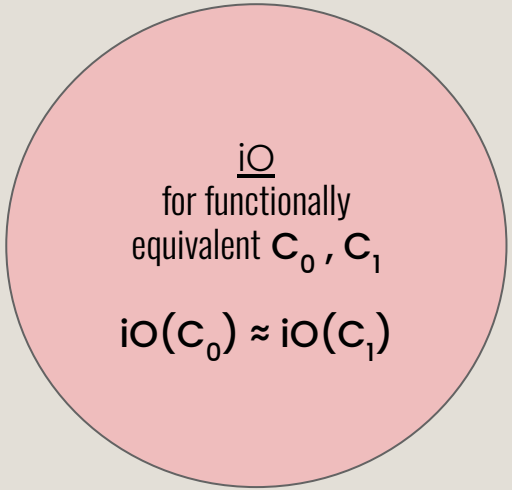


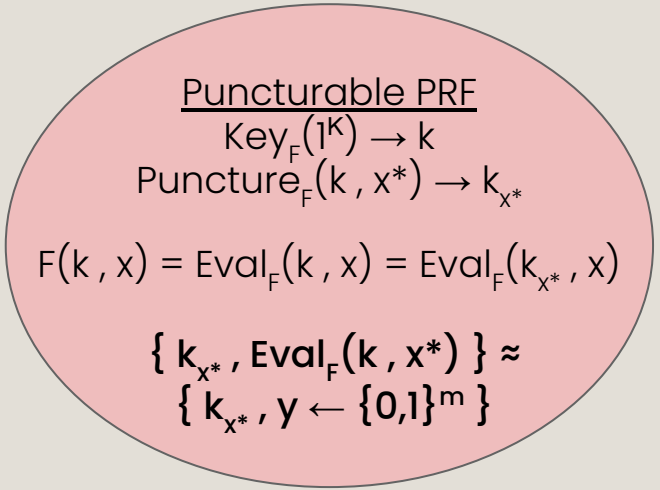
CONSTRUCTION



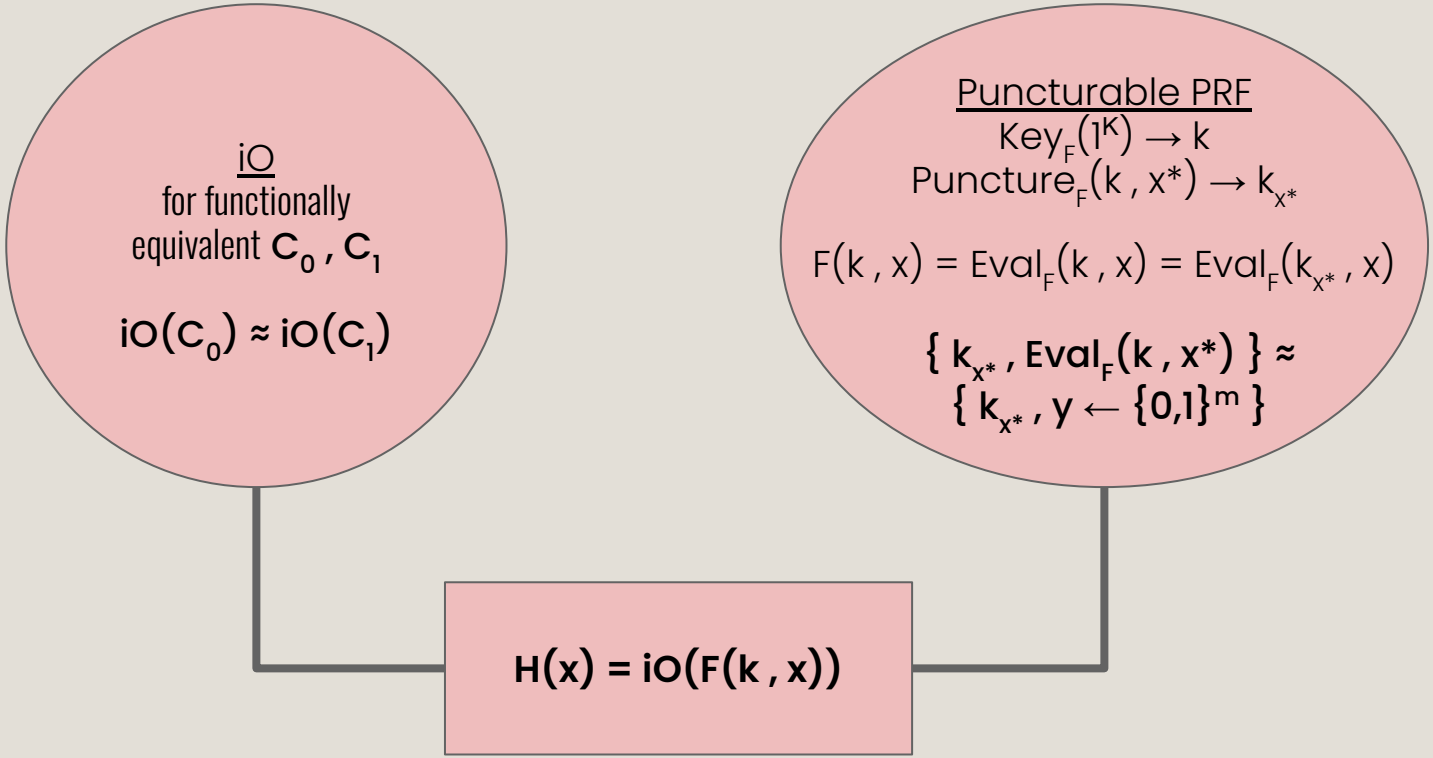


CONSTRUCTION





CONSTRUCTION



PROOF

PROOF

$H(\cdot) = iO(F(k, \cdot))$

$\text{QueryStrategy}^{\mathbf{CCR}}(1^K, r)$

$H(\cdot) = iO(F(k, \cdot))$

$\text{QueryStrategy}^{\mathbf{RO}}(1^K, r)$

PROOF

Hybrid 0

$H(\cdot) = iO(F(k, \cdot))$

$\text{QueryStrategy}^{\text{CCR}}(1^K, r)$

Hybrid 7

$H(\cdot) = iO(F(k, \cdot))$

$\text{QueryStrategy}^{\text{RO}}(1^K, r)$

PROOF

Hybrid 0

$H(\cdot) = iO(F(k, \cdot))$

$\text{QueryStrategy}^{\text{CCR}}(1^K, r)$

Hybrid 1

$\mathcal{O}_{\text{CCR}}$ outputs → random strings

$y^*_i = \text{QueryStrategy}(1^K, r, \{y^*_{i'}\}_{i'<i-1})$

$H^*(\cdot) = iO(F(k^*, \cdot))$

(programmed at outputs of $\mathcal{O}_{\text{CCR}}$)

Hybrid 7

$H(\cdot) = iO(F(k, \cdot))$

$\text{QueryStrategy}^{\text{RO}}(1^K, r)$

PROOF

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(programmed at outputs of $\mathcal{O}_{CCR}$)

Hybrid 7

$H(\cdot) = iO(F(k, \cdot))$

$\text{QueryStrategy}^{\text{RO}}(1^K, r)$

Hybrid 6

$y^*_i = \text{QueryStrategy}(1^K, r, \{y^*_{i'}\}_{i'<i-1})$

$H(\cdot) = iO(F(k, \cdot))$

(H independent of QueryStrategy outputs)

PROOF

PROOF

Hybrid 1

$F^*(x,i) = y_i^* \oplus b_i \Delta$

if $x = x_i \oplus \Delta$

(random strings y_i^*)

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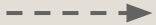

Hybrid 2

$F^*(x,i) = y_i^* \oplus b_i (x \oplus x_i)$

if $x \oplus x_i = \Delta$

(isolate Δ)

PROOF

Hybrid 1

$F^*(x,i) = y_i^* \oplus b_i \Delta$

if $x = x_i \oplus \Delta$

(random strings y_i^*)

Hybrid 2

$F^*(x,i) = y_i^* \oplus b_i (x \oplus x_i)$

if $x \oplus x_i = \Delta$

(isolate Δ)

Hybrid 3

$F^*(x,i) = y_i^* \oplus b_i (x \oplus x_i)$

if $\boldsymbol{\delta}_{\Delta}(x \oplus x_i) = 1$

(using point function $\boldsymbol{\delta}_{\Delta}$)

PROOF

Hybrid 1

$F^*(x,i) = y_i^* \oplus b_i \Delta$

if $x = x_i \oplus \Delta$

(random strings y_i^*)

Hybrid 2

$F^*(x,i) = y_i^* \oplus b_i (x \oplus x_i)$

if $x \oplus x_i = \Delta$

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Hybrid 3

$F^*(x,i) = y_i^* \oplus b_i (x \oplus x_i)$

if $\boldsymbol{\delta}_{\Delta}(x \oplus x_i) = 1$

(using point function $\boldsymbol{\delta}_{\Delta}$)

Hybrid 4

$F^*(x,i) = y_i^* \oplus b_i (x \oplus x_i)$

if $\boldsymbol{\delta}_{\boldsymbol{\pi}(\Delta)} \circ \boldsymbol{\pi}(x \oplus x_i) = 1$

(using hard-core predicate within $\boldsymbol{\pi}$)

PROOF

Hybrid 1

$F^*(x,i) = y_i^* \oplus b_i\ \Delta$

if $x = x_i \oplus \Delta$

(random strings y_i^*)

Hybrid 2

$F^*(x,i) = y_i^* \oplus b_i\ (x \oplus x_i)$

if $x \oplus x_i = \Delta$

(isolate Δ)

Hybrid 3

$F^*(x,i) = y_i^* \oplus b_i\ (x \oplus x_i)$

if $\boldsymbol{\delta}_{\Delta}(x \oplus x_i) = 1$

(using point function $\boldsymbol{\delta}_{\Delta}$)

Hybrid 4

$F^*(x,i) = y_i^* \oplus b_i\ (x \oplus x_i)$

if $\boldsymbol{\delta}_{\boldsymbol{\pi}(\Delta)} \circ \boldsymbol{\pi}(x \oplus x_i) = 1$

(using hard-core predicate within $\boldsymbol{\pi}$)

Hybrid 5

$F^*(x,i) = y_i^* \oplus b_i\ (x \oplus x_i)$

if $\boldsymbol{\delta}_{\boldsymbol{\pi}(\Delta)} \circ \boldsymbol{\pi}(x \oplus x_i) = 1$

(using the complement of the hard-core predicate within $\boldsymbol{\pi}$)

PROOF

Hybrid 1

$F^*(x,i) = y_i^* \oplus b_i \Delta$

if $x = x_i \oplus \Delta$

(random strings y_i^*)

Hybrid 2

$F^*(x,i) = y_i^* \oplus b_i (x \oplus x_i)$

if $x \oplus x_i = \Delta$

(isolate Δ)

Hybrid 3

$F^*(x,i) = y_i^* \oplus b_i (x \oplus x_i)$

if $\boldsymbol{\delta}_{\Delta}(x \oplus x_i) = 1$

(using point function $\boldsymbol{\delta}_{\Delta}$)

Hybrid 4

$F^*(x,i) = y_i^* \oplus b_i (x \oplus x_i)$

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Hybrid 5

$F^*(x,i) = y_i^* \oplus b_i (x \oplus x_i)$

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(using the complement of the hard-core predicate within $\boldsymbol{\pi}$)

Hybrid 6

$y^*_i = \text{QueryStrategy}(1^k, r, \{y^*_{i'}\}_{i'<i-1})$

$H(\cdot) = iO(F(k, \cdot))$

(H not programmed)

IMPLICATIONS and FUTURE WORK

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01

Feasibility Result:

Hash based free-XOR garbling schemes are secure even without assuming RO

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Hierarchy of Cryptographic Primitives:

Weak CCR $\not\Rightarrow$ collision resistance

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Feasibility Result:

Hash based free-XOR garbling schemes are secure even without assuming RO

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Hierarchy of Cryptographic Primitives:

Weak CCR $\not\Rightarrow$ collision resistance

03

Future Work:

de-obfuscation and efficient instantiation?

IMPLICATIONS and FUTURE WORK

01

Feasibility Result:

Hash based free-XOR garbling schemes are secure even without assuming RO

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Hierarchy of Cryptographic Primitives:

Weak CCR $\not\Rightarrow$ collision resistance

03

Future Work:

de-obfuscation and efficient instantiation?

04

Open Question:

Adaptive Security of hash-based free-XOR garbling schemes in the plain model

Thank You

https://eprint.iacr.org/2025/281